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A Risk Management System for Sustainable Fleet Replacement

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Abstract:

This article analyzes the fleet management problem faced by a firm when deciding which vehicles to add to its fleet. Such a decision depends not only on the expected mileage and tasks to be assigned to the vehicle but also on the evolution of fuel and CO_2 emission prices and on fuel efficiency. This article contributes to the literature on fleet replacement and sustainable operations by proposing a general decision support system for the fleet replacement problem using stochastic programming and conditional value at risk (CVaR) to account for uncertainty in the decision process. The article analyzes how the CVaR associated with different types of vehicle is affected by the parameters in the model by reporting on the results of a real-world case study.

Keywords: Decision Support Systems; Fleet Replacement; Risk Management; Stochastic Programming; Sustainable Operations; Conditional Value at Risk (CVaR).

1. Introduction

Sustainability is arguably the greatest challenge of our generation and the next. The question of how to meet the needs of the present without compromising the ability of future generations to meet their needs has important environmental, economic, and social dimensions (WCED, 1987). In today's global environment, tackling this challenge requires commitment from the private and the public sector, from nongovernmental organizations, and ultimately from all individuals. Due to these emerging concerns, companies are under serious pressure to evaluate their impact on the environment, to engage in evaluating the triple bottom line (people, profit, and planet), and consequently, to measure their resulting carbon footprint. Basic activities that contribute to this footprint are the production and transport of products, recycling, remanufacturing used products, and designing new products (Kleindorfer et al., 2005).

Fleet management is an important effort that will lead toward sustainable transportation in two ways. First, it has a direct economic effect on investment, maintenance, and operating costs. Second, it affects the resulting carbon footprint in the company. In addition, whereas the comparison of the relative cost of the different types of vehicles to consider in the fleet system is, in many aspects, a relatively obvious optimization problem, there exist additional complexities that make it an interesting research topic for sustainable management (see Figure 1). These include the uncertainties in market prices for various sources of energy, carbon emission prices, fuel consumption, and the mileage driven by the vehicles.

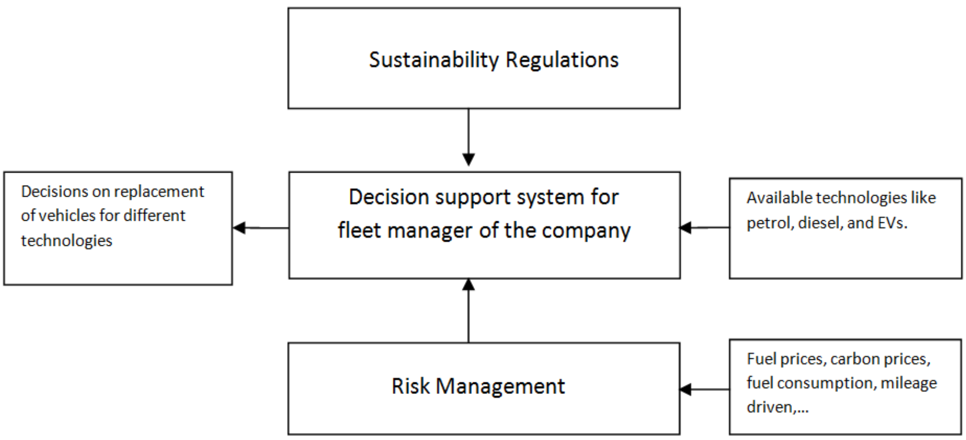

Figure 1. Decision support system for Fleet Manager

Under the traditional assumptions (Bethuyne, 1998; Chand et al., 2000; Hartman, 2001, 2004; Karabakal et al., 1994, 2000) the main concern for the fleet manager is to focus on the optimization of expected costs over a planning horizon. However, our approach is different from the standard literature on fleet replacement that does not consider the aforementioned risks and uncertainties in the problem.

This article considers the problem of leasing a vehicle during a given planning horizon, taking into account the uncertainties that exist in the real situations: carbon prices, fuel prices, mileage driven, and fuel consumption. The idea for this article arises from the need to study the possibility of adopting electrical vehicles (EVs) in many companies in Europe and in the US from an economic perspective. Because EVs are still in their infancy in terms of development and because they require a high investment cost, this study addresses this issue from a risk perspective, which has not been attempted earlier. The methodology that is used is a two-stage stochastic mixed integer programming model with conditional value at risk (CVaR). The main contribution of this work is to consider risk minimization as part of the objective function of the company.

The article is organized as follows: Section 2 presents the existing literature. Section 3 introduces the modeling of fleet replacement and develops a customized two-stage stochastic mixed-integer linear programming model (MILP) for minimizing risk and expected cost. Section 4 presents the analytical results on the comparison of the CVaR of different types of vehicles. Section 5 provides a case study for validating the analytical results, and section 6 describes the results of a real case study. Lastly, section 7 concludes the article.

2. Literature Review in Sustainable Fleet Replacement and Conditional Value at Risk

This article is based on two different, but complementary, bodies of literature. The first covers the area of sustainable operations, which includes the social and environmental impact of operations in the objective function of the companies that are maximizing profit. The second presents the literature on the conditional value at risk. The major part of the literature in sustainable operations concentrates on closed-loop supply chains, reverse logistics, and remanufacturing (Debo et al., 2005; Flapper et al., 2005; Savaskan et al., 2004). This article can be indirectly related to closed-loop supply chains in the sense that it focuses on justifying adoption of a green product (i.e., EVs) and the impact of CO_2 emissions on the supply chain and marketing strategy of automobile manufacturers (Atasu et al., 2008). Specifically, this section starts with a review of the current knowledge in the area of decision support systems for fleet management and then proceeds with a summary of the literature on the conditional value at risk.

2.1. Decision Support Systems for Sustainable Fleet Management

Decision support systems for fleet operations, capacity decisions, routing problems, and underlying optimization schemes are well developed in the logistics literature (e.g., Couillard, 1993; Ghiani et al., 2004; Lau et al., 2003). For instance, Ruiz et al. (2003) considered the vehicle routing problem, which has been widely studied in the literature. They introduced a new two-stage approach for solving a real problem along with decision-making software. In the first stage, all of the feasible routes are produced by means of an algorithm; subsequently, in the second stage, via an integer programming model, the optimum routes were chosen from the entire set of feasible routes.

Regarding the replacement strategies for fleet operations, the models generally can be categorized into two main groups based on different fleet characteristics: homogenous and heterogeneous models. In the homogeneous replacement model, a group of similar vehicles in terms of type and age that form a cluster (each cluster or group cannot be decomposed into smaller clusters) have to be replaced together. In contrast, in the heterogeneous model, multiple heterogeneous assets, such as fleets with different types of vehicles, have to be optimized simultaneously. For instance, vehicles of the same type and age may be replaced in different periods (years) because of the restricted budget for the procurement of new vehicles. The heterogeneous models are closer to the real-world commercial fleet replacement problem. These models are solved using integer programming and, generally, the input variables are assumed to be deterministic (Hartman, 1999, 2000, 2004; Karabakal et al., 1994; Simms et al., 1984). The methodology that has been most widely applied for solving homogenous models is dynamic programming. The advantage of the homogenous model is to assume probabilistic distributions for input variables in the optimization model (Bean et al., 1984; Bellman, 1955; Hartman, 2001; Hartman and Murphy, 2006; Oakford et al., 1984).

Another important classification of these models regards the nature of the replacement process: parallel vs. serial (Hartman and Lohmann, 1997; Jones et al., 1991). The main difference between parallel replacement analysis and serial replacement analysis is that the former takes into account how a policy exercised over one particular asset affects the rest of the assets in the same fleet. An example of parallel replacement would be a fleet of trucks that service a distribution center. In this case, the total capacity that is available is the sum of the individual capacities of the trucks. However, in a series replacement model, the assets operate in series, and consequently, demand is satisfied by the group of assets that operate in sequence. An example of this case is a production line in which multiple machines must work together to meet a demand or service constraint. In general, the capacity of the system is defined by the smallest capacity in the production line (Hartman, 2004). Our concern is related to parallel replacement models, which are used for the replacement plans of a set of fleets that are economically interdependent.

As an example of parallel replacement, Keles and Hartman (2004) study the fleet replacement policies for a city transit bus operator in Europe. The fleet includes over 600 buses, which provide service to a city in Europe with approximately 500,000 residents. Approximately 45,000 miles were accumulated in each year of operation. The main factors in their replacement decisions include the ability to select from multiple manufacturers while considering purchase price, government regulations, capital budgeting constraints, and economies of scale. They use an integer programming formulation for a parallel replacement problem in which there are multiple challengers available for replacement in each period.

Some additional examples of parallel replacement are the following. Sharma et al. (2007) use an MILP model to optimize decisions regarding leasing and logistics from the viewpoint of an electronic equipment leasing company. Moreover, they present a case study to validate their approach, providing a model for understanding the interaction of reverse logistics with equipment replacement. Hritonenko and Yatsenko (2012) analyze a fleet replacement problem under general assumptions about technological development. They provide an optimization model that takes into account the variable lifetime of assets in a deterministic infinite-horizon framework. The optimal dynamics of the asset lifetime and investment is obtained under situations involving technological change and technological shocks that influence the operating and new asset costs. Figliozzi et al. (2011) developed a model for considering the economic and environmental optimization of vehicle replacement decisions from a fleet manager's viewpoint. They introduce an integer programming vehicle replacement model that is used to evaluate the current environmental and governmental intervention issues, such as greenhouse gases, taxes, and fiscal incentives for EVs purchases. Moreover, they take into account the effect of utilization (mileage per year per vehicle) and petrol prices on fleet purchasing decisions. Although the machine or vehicle replacement literature is rich in models dealing with budget constraints (Chand et al., 2000; Karabakal et al., 1994, 2000), variable utilization (Bethuyne, 1998), stochastic demands (Hartman, 2001), and heterogeneous types of vehicles (Hartman, 2004), these models have not considered risk management. In summary, from this literature review, it is evident that there is a gap that this article aims to address: to explain sustainable fleet replacement from an uncertainty perspective using risk management methodologies.

2.2. Stochastic Modeling with Discrete Scenarios and CVaR

Stochastic decision models depend on future events. The future can be represented by a set of scenarios that are discrete realizations of stochastic parameters. The method used for obtaining the discrete outcomes of a random parameter is referred to as "scenario tree generation". There are different ways to generate scenario trees, such as simulation, clustering, and optimization techniques (Hoyland et al., 2003). Discretization of the random values and the probability space leads to a framework in which a random variable takes a finite number of values. Thus, the factors driving the risky events are approximated by a discrete set of scenarios or a sequence of events. This branching process is represented by means of a scenario tree. The root node in the scenario tree represents "today" and is immediately observable from deterministic data. The nodes that come after represent the events of the world, which are conditional at later stages. The arcs linking the nodes represent various realizations of the uncertain variables. An ideal situation is that a generated set of scenarios represents the whole universe of possible outcomes of the random variable.

Quaranta et al. (2008) mention that the major problem related to variance as a measure of risk measurement is that it takes into account the upside and downside of distributions equally. As a result, financial specialists typically focus on quantile-based measures, such as value at risk (VaR). The definition of VaR is the minimum potential loss that a financial sector can tolerate with a certain likelihood during a finite period. However, VaR, if considered in the framework of coherent risk measures, lacks subadditivity and, consequently, convexity (e.g., Artzner et al., 1997) for general distributions (although it may be subadditive for special cases, e.g., for normal distributions). To solve these problems, recent literature has focused on coherent risk measures such as CVaR, e.g., Rockafellar, and Uryasev (2000, 2002). Moreover, other coherent risk measures in the case of asymmetric asset distributions (Goh et al., 2012) and a robust optimization approach for CVaR (e.g., Chen et al., 2010) have also been mentioned.

Briefly, CVaR is defined as the conditional expectation of the losses beyond VaR. Indeed, in contrast to VaR, CVaR provides extra information on the losses in the tail of the loss distribution beyond VaR (Figure 2).

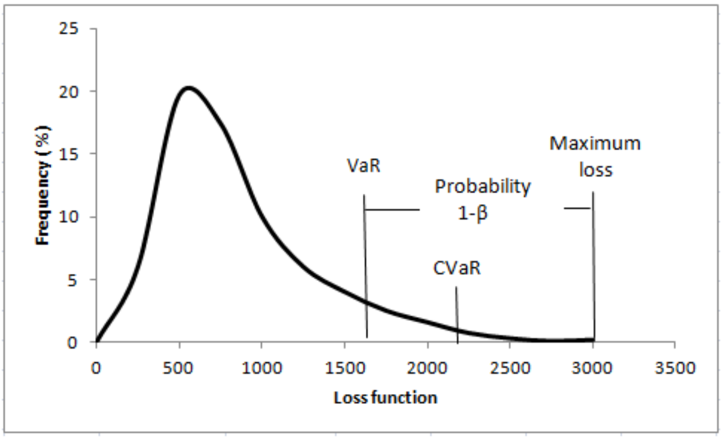

Figure 2: VaR, CVaR, and Maximum loss, Rockafellar and Uryasev, 2000

CVaR is a consistent measure of risk because it is subadditive and convex (Artzner et al., 1999). Moreover, it has been proven that it can be optimized using linear programming, which can handle portfolios with a very large number of scenarios (Rockafellar and Uryasev, 2000). In addition, minimization of CVaR leads to near optimal solutions for VaR, and when the return-loss distribution is normal, these two risk measures produce the same optimal portfolio (Rockafellar and Uryasev, 2000). The linear program model suggested by Rockafellar and Uryasev for simultaneous minimization of CVaR and calculation of VaR is as follows:

$$\min_{x\in X, z\in R, \alpha_\beta \in R} \phi_\beta = \min_{x\in X, z\in R\ ,\alpha_\beta \in R} \alpha_\beta + v\sum_{q=1}^{Q} z_q$$

$$s.t.\ \ x \in \mathrm{X},$$

$$\mathrm{z}_q \geq f(x,\gamma_q) - \alpha_\beta, \quad \mathrm{z}_q \geq 0, \quad q = 1,\ldots Q$$

In the above model, ϕ_β and α_β denote the CVaR and VaR for the confidence level of *β,* respectively. In addition, *q* represents the number of scenarios, and γ_q shows the vector of stochastic variables in scenario *q* sampled from the distribution of the stochastic processes in the model, $v = ((1\text{-}\beta)\ \mathrm{Q})^{-1}$, where *x* is the vector of decision variables, z_q are positive dummy variables, and *f* denotes the loss function. Solving the above LP model simultaneously yields the optimal value of ϕ^*_β, the decision variable, which is x^*, and α^*_β.

3. Modeling the Fleet Replacement Policy

This section presents a stochastic model for vehicle leasing in a given planning horizon. The aim is to obtain the optimal policy that minimizes the cost and the risk simultaneously. Because equation (1) uses a two-stage model, the average fuel prices (f_u) are calculated. In equation (1), f_{ut} denotes the forecasted fuel prices during the planning horizon, and T is the length of the horizon in years. Moreover, equation (2), to obtain the average electricity charge costs of batteries of EVs, e_{bu}, (for different brands with respect to a benchmark brand for 100 miles) uses the average electricity prices from equation (1). In equation (2), p_b is the ratio of battery capacity to the benchmark brand (the notation is summarized in Table 1).

$$f_u = \sum_{t=1}^{T} f_{ut} / T \qquad \text{For all } u \qquad (1)$$

$$e_{bu} = f_u p_b \qquad \text{For all } u \text{ and } b \qquad (2)$$

Next, equation (3) computes the running cost for each brand of fossil fuel technology per 100 km under different scenarios. In addition, the running cost for electric vehicles (EVs) per 100 km is calculated using equation (4). In equation (3), o_{bv} denotes fuel consumption for fossil fuel vehicles per 100 km. The cost of CO_2 emissions for different technologies in (3) and (4) is taken into account by including parameter c_p^s, which shows carbon prices in different states of the world. Other parameters in (3) and (4) are described in Table 1.

$$r_{ibusv} = o_{bv}(f_u + c_p^s c^g / 10^6) \qquad \text{For all } b, u, s, v, \text{ and } i=1 \qquad (3)$$

$$r_{ibusv} = e_{bu} / W + 100(c^e / 10^6)c_p^s \qquad \text{For all } b, u, s, v, \text{ and } i=2 \qquad (4)$$

Consequently, equation (5), based on equations (3) and (4), computes the total running cost over the planning horizon. In equation (5), D_m represents the monthly mileage driven by a vehicle.

$$y_{ibusmv} = 48(r_{ibusv} / 100)WD_m \qquad \text{For all } i, b, u, s, v, \text{ and } m \qquad (5)$$

Lastly, equations (6) and (7) calculate the total investment (fixed) cost for fossil fuel technologies (6) and for EVs (7). The monthly leasing cost, which is shown by l_{ib}, is used to obtain total fixed costs in the planning horizon. Moreover, for EVs, there is an extra investment cost, which is the annual lease cost for batteries, B_b.

Table 1. The indices, decision variables, and parameters for technologies, brands, and different scenarios for carbon and fuel prices, mileage driven, and fuel consumption

i=1, 2 index for fossil fuel and electrical technologies, respectively

b=1, 2, B index for brands for b_1, b_2, and benchmark brand (b_B), respectively

t=1, 2, .., T index for number of periods in the year from the beginning of 2012 to 2016

s=1, 2, S index for the state of carbon prices for low, medium, and high, respectively

u=1, 2,.., U index for the forecasted fuel price scenarios from of 2012 to 2016

m=1, 2,.., M index for scenarios from the distribution of monthly mileage driven by a car

v=1,….,V index for scenarios from the distribution of fuel consumption by a car

W: Conversion coefficient of mileage to km

ω: Parameter for trade-off of risk and cost in the objective function

β: Confidence level for calculating CVaR and VaR

x_{ib}: The car with technology i and brand b that has been leased

z_{ibusmv}: Auxiliary stochastic variables for the loss function

α_{ib}^{β}: Value at risk at confidence level β for a car with technology i and brand b

ϕ_{ib}^{β}: Conditional value at risk at confidence level β for a car with technology i and brand b

B_b: Annual lease cost for batteries of EVs with brand b

p_b: Ratio of the capacity of the battery of EVs with brand b to the benchmark brand (22 kw)

c_p^s : The expected CO_2 prices for each state s

c^e: The CO_2 emissions (gr) per km for electrical technology

c^g: The CO_2 emissions (gr) per liter for fossil fuel technology

l_{ib}: The monthly lease cost for each technology i with brand b.

D_m: The monthly mileage driven by a car for each scenario m

o_{bv}: Fuel consumption per 100 km for brand b and each scenario v

f_{ut}: Forecasted fuel prices for each scenario u in year t

f_u: Average fuel prices for each scenario u during the planning horizon

$e_{bu:}$ Average charge cost of EV batteries for 100 miles with brand b for each scenario u

r_{ibusv}: The running cost per 100 km for technology i with brand b for each scenario u, each state of carbon price s, and each scenario for fuel consumption v

y_{ibusmv}: The total running cost per technology i with brand b for scenario u, each state of carbon price s, each scenario for monthly mileage driven by cars m, and each scenario for fuel consumption v

μ_{ib}: The total fixed cost per technology i with brand b

$$\mu_{ib} = 48 l_{ib} \quad \text{For all } b \text{ and } i\text{=1} \tag{6}$$

$$\mu_{ib} = 48 l_{ib} + 4 B_b \quad \text{For all } b \text{ and } i\text{=2} \tag{7}$$

The objective is to minimize the weighted average of CVaR and the total expected cost. The decision variable is x_{ib}, which denotes a vehicle with technology i and brand b. By combining the formulas and parameters presented in the previous sections, the stochastic mixed integer programming problem is represented by equations (8)-(13).

$$\underset{x\in\{0,1\}, z\in R, \alpha_{ib}^{\beta}\in R}{Min} = \omega\, E(\text{cost}) + (1-\omega)\phi_{ib}^{\beta} \tag{8}$$

$$E(\text{cost}) = \sum_{i=1}^{2}\sum_{b=1}^{B}\mu_{ib}x_{ib} + \left(\sum_{i=1}^{2}\sum_{b=1}^{B}\sum_{u=1}^{U}\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{v=1}^{V} y_{ibusmv}x_{ib}\right) / USMV \tag{9}$$

$$\phi_{ib}^{\beta} = \alpha_{ib}^{\beta} + 1/(USMV(1-\beta))\sum_{u=1}^{U}\sum_{s=1}^{S}\sum_{m=1}^{M}\sum_{v=1}^{V} z_{ibusmv} \quad \text{For all } i \text{ and } b \tag{10}$$

$$z_{ibusmv} \geq (\mu_{ib} + y_{ibusmv})x_{ib} - \alpha_{ib}^{\beta} \quad \text{For all } i, b, u, s, m, \text{ and } v \tag{11}$$

$$\sum_{i=1}^{2}\sum_{b=1}^{B} x_{ib} = 1 \tag{12}$$

$$x_{ib} \in \{0,1\} \tag{13}$$

Because the objective of this stochastic program is to minimize the cost and risk simultaneously, equation (8) minimizes the weighted average of the total expected cost, E (cost), and CVaR. That is, by changing the value of parameter ω to different combinations of the total expected cost, the risks over the planning horizon are minimized, depending on whether the focus is more on cost or on risk. Equation (9) calculates the expected total cost, which includes the fixed cost and running cost. The running cost is calculated based on the realization of all of the stochastic processes for each brand and technology. Moreover, equations (10)-(11) compute the value of CVaR at confidence level β (Rockafellar and Uryasev, 2000). In inequality (11), the first term on the right-hand side denotes the loss function (Rockafellar and Uryasev, 2000), and it is related to the total expected cost for different scenarios. Lastly, equations (12)-(13) are the constraints on the decision variable. Solving (8)-(13), depending on the value of ω, yields the optimal vector x^*, corresponding VaR*, optimal CVaR*, and total expected cost.

4. Analytical Results on the Comparison of the CVaR of Different Technologies

This section presents the analytical results comparing the CVaR of different technologies. Let y_{ibus} denote the stochastic total running cost for technology *i* with brand *b*, taking into account *u* scenarios for fuel prices and *s* states for carbon prices.

Proposition 1*: The* $\phi^{\beta}(y_{2bus})$ *for EVs is less than the* $\phi^{\beta}(y_{1bus})$ *for fossil fuel vehicles if and only if this condition holds:* $\phi^{\beta}(f_u) - \phi^{\beta}(e_{bu}) / (Wo_b) \geq c_p{}^s (c^g / 10^6 - c^e / (o_b 10^4))$.

Proof: As $\phi^{\beta}(.)$ is a coherent risk measure (Artzner et al., 1999):

$$\phi^{\beta}(hX) = h\phi^{\beta}(X) \tag{14}$$

$$\phi^{\beta}(X+h) = \phi^{\beta}(X) - h \tag{15}$$

$$\phi^{\beta}(X+Y) \leq \phi^{\beta}(X) + \phi^{\beta}(Y) \tag{16}$$

In equations (14)-(16), *h* is an arbitrary constant and *X* and *Y* denote stochastic variables. Moreover, these equations are referred to as the Positive Homogeneity, Translation Invariance, and Subadditivity properties of coherent risk measures, respectively (Artzner et al., 1997). The comparison of the $\phi^{\beta}(.)$ for EVs with fossil fuel vehicles is based on the stochastic total running cost. Based on equations (3)-(5) and (14)-(15), equation (17) can be derived.

$$\phi^{\beta}(y_{1bus}) = 0.48WDo_b\phi^{\beta}(f_u) - 0.48o_bWDc_p{}^s c^g / 10^6 \qquad \text{For all } b, u, s \tag{17}$$

Specifically, equation (17) is for the case considering fossil fuel prices as stochastic processes. In equation (17), *D* denotes the expected monthly mileage driven by a car and is a constant parameter (Table 4). Moreover, for the case of EVs in equation (18) using properties (14)-(15), the value of $\phi^{\beta}(.)$ is calculated.

$$\phi^{\beta}(y_{2bus}) = 0.48D\phi^{\beta}(e_{bu}) - 48WDc_p{}^s c^e / 10^6 \qquad \text{For all } b, u, s \tag{18}$$

By subtracting (18) from (17), which is denoted by *k*, the relationship in (19) is obtained.

$$0.48WDo_b\phi^{\beta}(f_u) - 0.48D\phi^{\beta}(e_{bu}) - 0.48o_bWDc_p{}^s c^g / 10^6 + 48WDc_p{}^s c^e / 10^6 \geq 0 \tag{19}$$

By dividing both sides of inequality (19) by $0.48WD$, it follows that $\phi^{\beta}(f_u)-\phi^{\beta}(e_{bu})/(o_bW)\geq c_p{}^s(c^g/10^6-c^e/(o_b10^4))$. Then, setting $k'=c^g/10^6-c^e/(o_b10^4)$ yields (20).

$$\phi^{\beta}(f_u)-\phi^{\beta}(e_{bu})/(o_bW)\geq c_p{}^s k'. \quad \blacksquare \qquad (20)$$

The right-hand side of inequality (20) is a positive number. The left-hand side is a stochastic variable because its value depends on the various realizations of fuel prices in different scenarios. To illustrate, the figure represents inequality (20).

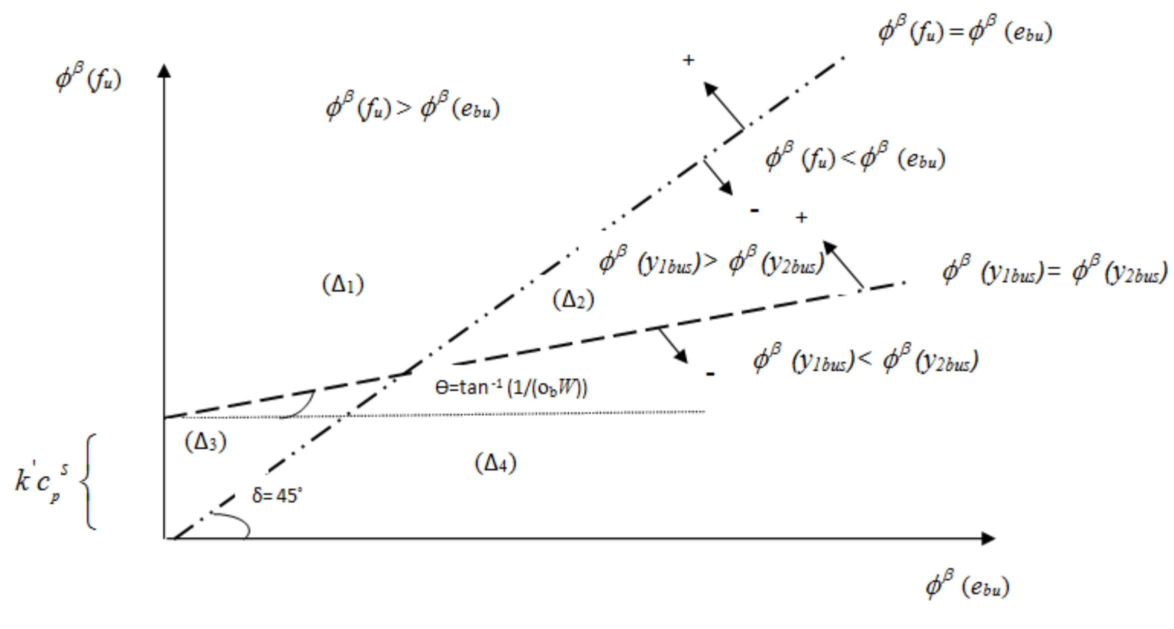

Figure 3. The feasible solution for proposition 1, which is shown by (Δ_1) and (Δ_2)

As can be seen in Figure 3, there are four areas in which the relationship between different values of $\phi^{\beta}(.)$ for different stochastic processes is presented. Specifically, the feasible solution area that is shown by (Δ_1) and (Δ_2) is the area in which condition (20) holds. However, in the parts that are shown by (Δ_3) and (Δ_4), there is no feasible solution for inequality (20). The interesting point about area (Δ_2) is that the value $\phi^{\beta}(.)$ for fossil fuel prices is smaller than that for electricity prices, but it is higher for the stochastic total running cost. The reason is the different slopes and positive number ($k'c^s{}_p$), which are shown in Figure 3. As a result, in these areas, the value of k would be positive, and $\phi^{\beta}(y_{2bus})$ for EVs is less than the $\phi^{\beta}(y_{1bus})$ for fossil fuel vehicles.

Let y_{ibsm} denote the stochastic total running cost for technology *i* with brand *b*, taking into account *m* scenarios for mileage driven and *s* states for carbon prices, and let *f* denote the average fuel price during the planning horizon.

Proposition 2: *The* $\phi^{\beta}(y_{2bus})$ *for EVs is less than the* $\phi^{\beta}(y_{1bsm})$ *for fossil fuel vehicles if and only if this condition holds*: $e_b - Wo_b f < c_p{}^s Wo_b (c^g / 10^6 - c^e / (o_b 10^4))$.

Proof: The stochastic total running cost in which stochastic parameters exist is used to compare $\phi^{\beta}(.)$ for the EVs and for the fossil fuels. Therefore, based on equations (3)-(5) and property (14), it follows that, for all *b, m, s*:

$$\phi^{\beta}(y_{1bsm}) = \phi^{\beta}(0.48 r_{1bs} W D_m) = 0.48 W r_{1bs} \phi^{\beta}(D_m) = 0.48 W \phi^{\beta}(D_m)(o_b f + o_b c_p{}^s c^g / 10^6).$$
(21)

Specifically, equation (21) holds for fossil fuel vehicles, considering mileage driven as the stochastic process. Moreover, for the case of EVs, the value of $\phi^{\beta}(.)$ is obtained using property (14) and represented by equation (22), for all *b, m, s*:

$$\phi^{\beta}(y_{2bsm}) = \phi^{\beta}(0.48 r_{2bs} W D_m) = 0.48 W r_{2bs} \phi^{\beta}(D_m) = 0.48 W \phi^{\beta}(D_m)(e_b / W + 100 c_p{}^s c^e / 10^6) \quad (22)$$

Therefore, by comparing (22) and (21), we derived the inequality (23).

$$\phi^{\beta}(y_{2bsm}) < \phi^{\beta}(y_{1bsm}) \Leftrightarrow e_b - Wo_b f < c_p{}^s Wo_b k' . \quad \blacksquare \quad (23)$$

In inequality (23), both sides are real numbers depending on different values of parameters and the values of *f* and e_b, which are the expected values of fuel prices and electricity prices, respectively. For illustration, inequality (23) is represented in Figure 4. The feasible solution areas are represented by (Δ_1) and (Δ_2). Indeed, in these areas, the condition in proposition (2) is true, and $\phi^{\beta}(y_{2bsm})$ for EVs is less than the $\phi^{\beta}(y_{1bsm})$ for fossil fuel vehicles.

Let y_{ibsv} denote the stochastic total running cost for technology *i* with brand *b*, taking into account *v* scenarios for fuel consumption and *s* states for carbon prices.

Proposition 3: *The* $\phi^{\beta}(y_{2bsv})$ *for EVs is less than the* $\phi^{\beta}(y_{1bsv})$ *for fossil fuel vehicles if and only if this condition holds:* $\phi^{\beta}(o_{bv}) > \dfrac{(e_b / W + c_p{}^s c^e / 10^4)}{(f + c_p{}^s c^g / 10^6)}$.

Proof: The stochastic total running cost is used to compare the $\phi^{\beta}(.)$ of EVs with the $\phi^{\beta}(.)$ of EVs of fossil fuel vehicles. Based on equations (3)-(5) and properties (14) and (16), inequality (24) is obtained.

$$\phi^{\beta}(y_{1bsv}) \leq 0.48 WD \phi^{\beta}(o_{bv})(f + c_p{}^s c^g / 10^6) \qquad \text{For all } b, v, s \tag{24}$$

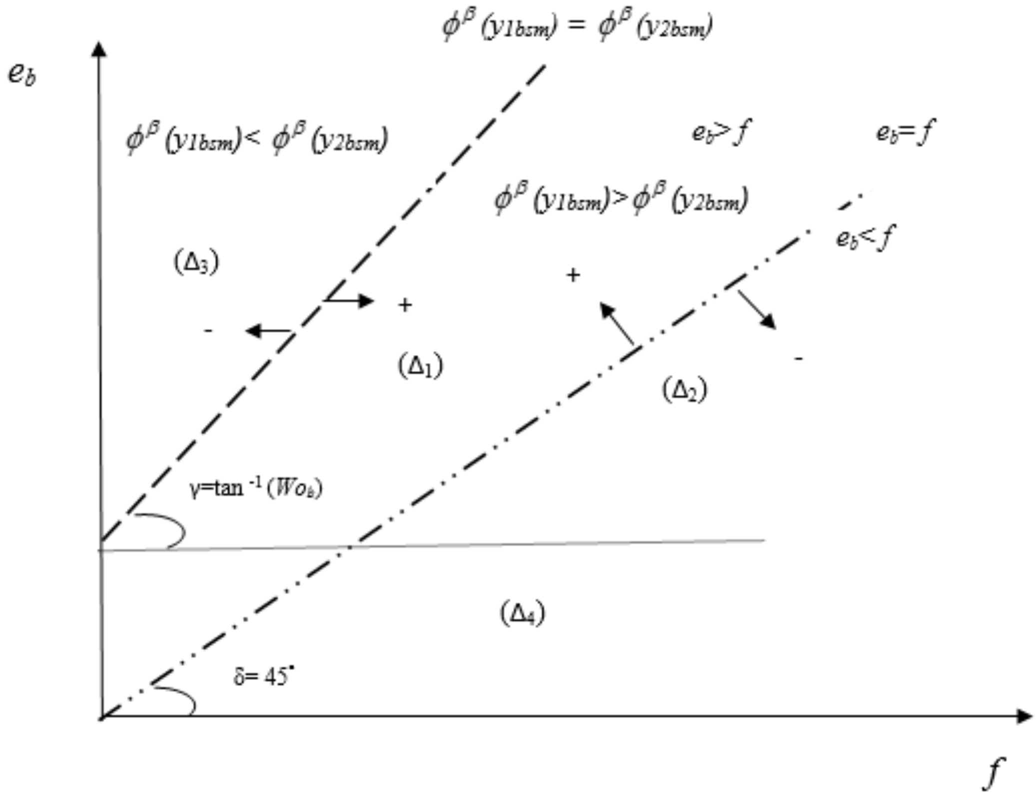

Figure 4. The feasible solution for proposition 2, which is shown by (Δ_1) and (Δ_2).

Inequality (24) is obtained based on the subadditivity property (16), and it holds for fossil fuel vehicles when considering fuel consumption as the stochastic process. In addition, for the case of EVs, the value of $\phi^{\beta}(.)$, represented by equation (25), is obtained from property (14). So by comparing (24) and (25), after some basic algebra, inequality (26) is derived. The right-hand side of inequality (26) is a positive number, between one and two, depending on the different values of parameters and the values of f and e_b, which are expected values of fuel prices and electricity prices, respectively. However, the left-hand side is a stochastic variable depending on the realization of different scenarios for the fuel consumption of fossil fuel technologies per 100 km. Inequality (26) is illustrated in Figure 5.

$$\phi^{\beta}(y_{2bsv}) = 0.48WD(e_b / W + 100c_p{}^s c^e / 10^6) \qquad \text{For all } b, v, s \qquad (25)$$

$$\phi^{\beta}(o_{bv}) > \frac{(e_b / W + c_p{}^s c^e / 10^4)}{(f + c_p{}^s c^g / 10^6)} \; \blacksquare \qquad (26)$$

Because f and e_b are correlated, $\phi^{\beta}(o_{bv})$ is represented as a function of f. As represented in Figure 5, the graph is a decreasing homographic function with a horizontal asymptote k'' equals to $e_b /(Wf)$. The intuition behind this pattern is that by increasing the expected fuel prices, the $\phi^{\beta}(o_{bv})$ will decrease due to lower fuel consumption. Specifically, the feasible solution area, which is represented by (Δ_1) and (Δ_2), is the area in which condition (26) holds. Therefore, it follows that the $\phi^{\beta}(y_{2bsv})$ for EVs is less than the $\phi^{\beta}(y_{1bsv})$ for fossil fuel vehicles.

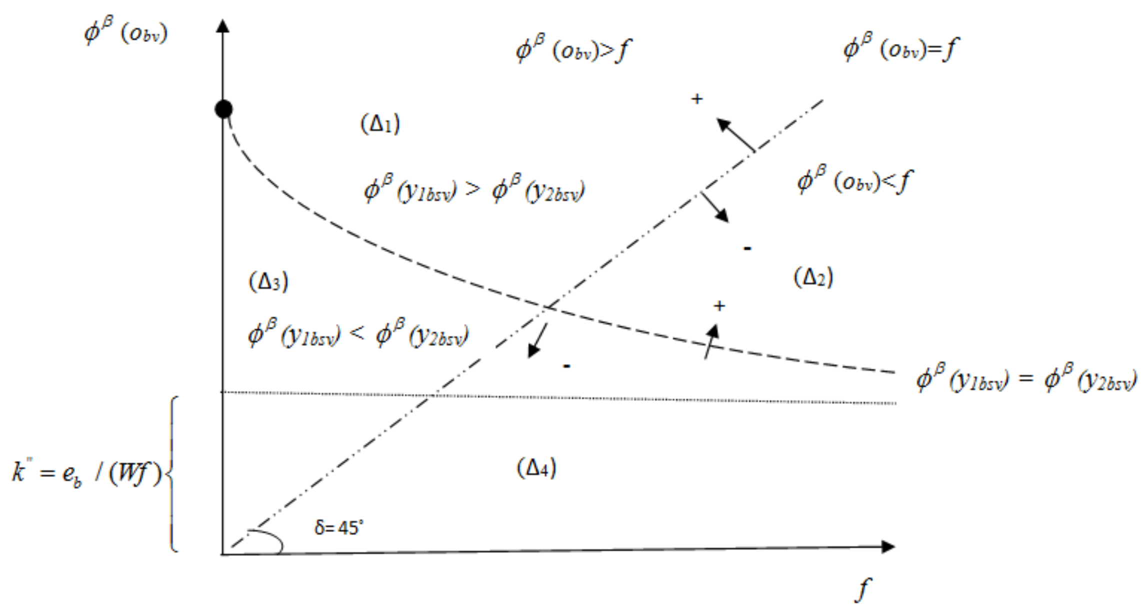

Figure 5. The feasible solution for Proposition 3, which is represented by (Δ_1) and (Δ_2)

5. A Case Study on Sustainable Fleet Management

An important issue when developing a model is to determine whether it is an accurate representation of the system studied, i.e., if it is valid (e.g., Landry et al., 1983; Law and Kelton, 1991; Landry and Oral, 1993). The term "accurate representation" is used to mean the extent to which the model fits the real system either in terms of structure and mechanism or in terms of output, depending on the context of the problem. In this article, the validity of the model was ensured in using the following steps. The case presented in this section was based on: 1) real data from the fleet analyzed, including mileage and consumption per vehicle; 2) real data on the leasing costs for different types of vehicles and brands; 3) forecasts for the fuel prices for the planning horizon considered, based on real data; and 4) a model for CO_2 prices estimated from real data.

Moreover, the validity of the model is also tested by comparing the decisions recommended by the model with the current fleet used by the company. This is reported in section 6 for the case in which only expected values were used: in this case, as is currently the case, the optimal decision is to lease diesel vehicles only.

The goal is to obtain the optimal policy for vehicle replacement, using leasing, by considering a planning horizon of four years (2012 to the beginning of 2016). Three fuel technologies (Petrol, Diesel, and Electricity) and three brands (b_1, b_2, and b_B,) are considered; b_B is the benchmark. Even though they are based on real vehicles, for the purpose of anonymity, the brands are denoted as such. The typical consumption of the benchmark brand is 7.6 liters/100 km for diesel and 9.3 liters/100 km for petrol. A current petrol price of approximately £1.37/liter and diesel price of £1.41 /liter are assumed. The cost for leasing the battery of the electric vehicle, for the benchmark brand, is £ 950 per year in the UK, and the cost to charge is £2.5 per charge (for 100 miles autonomy). (In this study, electricity and electricity charge prices are used interchangeably. However, indeed, the price of electricity is the price of each charge for the 22 kWh battery of the benchmark brand).

Therefore, to obtain the electricity charge for other brands, the ratio of the power of the battery with respect to the battery of the benchmark brand (Table 2) can be used. The emissions in the UK are estimated to be approximately 81 g/km (benchmark's estimate), and for petrol and diesel, the emissions are estimated to be approximately 2310 and 2680 g/liter, respectively. Moreover, for carbon prices, because there is no clear historical trend, three states of prices (low prices, £5, medium prices, £10, and high prices, £20) are used. The other parameters for other brands, including the benchmark brand are presented in Tables 2 and 3.

Table 2. The parameters for electric vehicles with different brands

Brand	Cost of renting the battery per year(£)	The ratio of the battery of each brand to the benchmark brand (22 kw)
b_1	1100	1.6
b_2	1050	1.3
b_B	950	1

Given all of the assumptions about the electric version of the benchmark brand, it follows that they are less competitive in comparison with the diesel and petrol version of it when the annual expected mileage driven is less than 19526 miles/year, with the last assumptions and monthly leasing costs of £220, £230, and £380 for petrol, diesel, and EVs, respectively. As represented in Figure 6, the total costs, including the running and investment cost for EVs, with last assumptions about the benchmark brand's parameters, are less than other technologies when the total annual mileage is above the intersection of the diesel and electric lines. Moreover, petrol cars are more competitive in comparison with the other two technologies when the average annual mileage driven is less than 2843 miles/year. For diesel cars, it is economical to use them when the average annual mileage is between 2843 and 19526 miles. These thresholds depend on fuel and carbon prices and monthly leasing costs.

Table 3. The leasing costs and fuels consumption for cars with different brands

Technology/Brand	Monthly lease cost (£)	Fuel consumption per 100 km (liter)
Petrol-b_1	230	9.8
Diesel-b_1	240	7.9
Electric-b_1	450	No fuel consumption
Petrol- b_2	210	9.1
Diesel- b_2	220	6.9
Electric-b_2	400	No fuel consumption
Petrol-b_B	220	9.3
Diesel-b_B	230	7.6
Electric-b_B	380	No fuel consumption

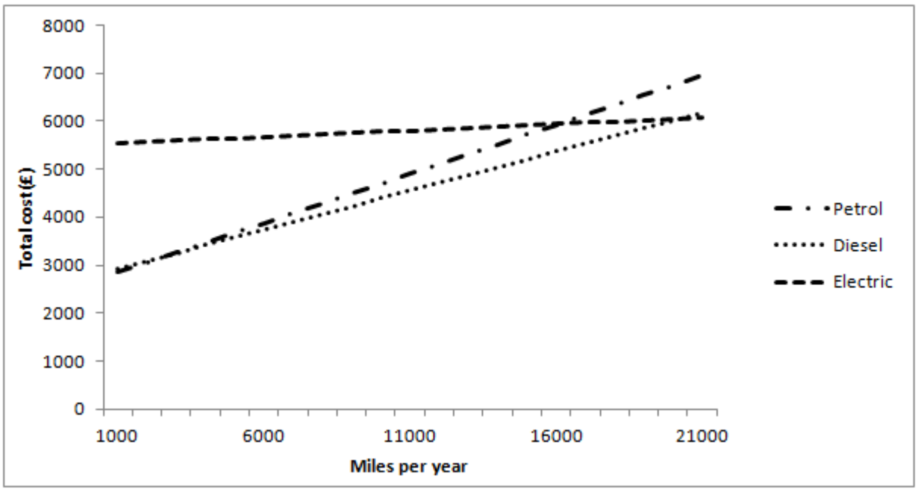

Figure 6: The total cost (running plus investment cost) versus the average mileage driven in one year for different technologies.

5.1. Vector Auto Regression for Forecasting Fuel Prices

The historical data used for fuel prices is based on a time series from Jan. 2000 to Dec. 2011. Because the fuel prices are correlated (Table 4), the method used in this article to forecast fuel prices is Vector Auto Regression. The Vector Auto Regression is used in forecasting systems of interrelated time series for analyzing the dynamic impact of random disturbances on the system of variables. The Vector Auto Regression approach treats every endogenous variable as a function of the lagged values of all of the endogenous variables in the system. The mathematical representation of Vector Auto Regression is the following:

$$Y_t = A_1\, Y_{t-1} + \ldots + Ap\, Y_{t-p} + BX_t + e_t$$

where Y_t is a vector of endogenous variables, X_t is a vector of exogenous variables, A_1, A_2, …, A_p and B are matrices of coefficients to be estimated, and e_t is a vector of white noises that may be contemporaneously correlated but are uncorrelated with their own lagged values and uncorrelated with all of the right-hand side variables.

Because fuel prices are not stationary, first-order differentiation is used to convert them to a stationary process (e.g., Akaike, 1977; Fuller, 1976; Lütkepohl, 1991; Schwarz, 1978). Please refer to the Online Appendix for the details of the estimation of equations (27)-(29), in which p_t, d_t, and e_t denote petrol, diesel, and electricity prices at time t. In these equations the fuel price is described as a function of the significant lagged values of two other fuel prices and its own white noise (ε_t) that are uncorrelated with their own lagged values and all of the right-hand side variables. Moreover, these equations show that by considering two additional fuel prices as endogenous variables in the main equation for forecasting each of them, the correlation between the fuel prices is taken into account (Table 4). These equations are used in fuel price forecasting over the planning horizon.

$$p_t = 0.004 + p_{t-1} - 0.211(e_{t-1} - e_{t-2}) + \varepsilon_{tp} \quad (27)$$

$$d_t = 0.003 + 1.506 d_{t-1} - 0.506 d_{t-2} - 0.12(e_{t-1} - e_{t-2}) + \varepsilon_{td} \quad (28)$$

$$e_t = 0.006 + 1.354 e_{t-1} - 0.354 e_{t-2} + \varepsilon_{te} \quad (29)$$

$$\varepsilon_{tp} = N(0,\sigma_p),\ \varepsilon_{td} = N(0,\sigma_d),\ \varepsilon_{te} = N(0,\sigma_e)$$

Table 4. Correlation matrix for fuel prices from Jan. 2000 to Dec. 2011

	Petrol	Diesel	Electric
petrol	1.00	0.99	0.85
Diesel	0.99	1.00	0.88
Electric	0.85	0.88	1.00

5.2. Modeling Uncertainty about the Driven Mileage

As presented in Figure 7, in the dataset of approximately 2789 vehicles, the monthly mileage driven follows a lognormal distribution with the mode at approximately 500 miles per month. In this sample, 9.2% of the vehicles had zero mileage during the period analyzed.

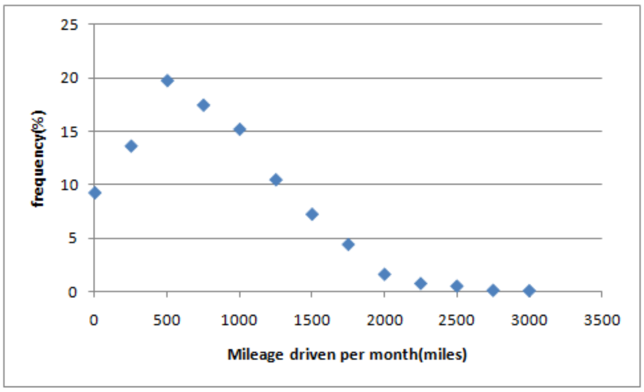

Figure 7. Actual distribution of monthly mileage driven by cars based on the data.

As seen in Table 5 the mean mileage is approximately 834 miles per month, and the median is 750 miles per month. This result implies that 50% of the vehicles are used less than 750 miles per month. For considering the mileage driven by a car, the scenarios are generated using a lognormal distribution with its parameters estimated to fit the data (Evans et al., 2000).

Table 5. Descriptive statistics for the monthly mileage driven by cars

Mileage driven per month	Miles
Mean	834.27
Mode	500.00
Median	750.00
Std. Deviation	510.17

5.3. Modeling Uncertainty about Fuel Consumption

Another stochastic parameter that is considered in the analysis is fuel consumption per 100 km, both by diesel and petrol cars. Because fuel consumption depends on the different conditions under which the vehicles are used (e.g., motorways vs. urban areas) and the skill of the driver, it is essential to consider it as a stochastic process. As shown in Figure 8, the fuel consumption per 100 km, based on the real data used in the present study, follows a lognormal distribution. In this case, our data include 2789 diesel vehicles with a consumption mode at approximately 5 liters/100 km; in the period under analysis, 13.6% of the vehicles had an average consumption of approximately 4 liters per 100 km.

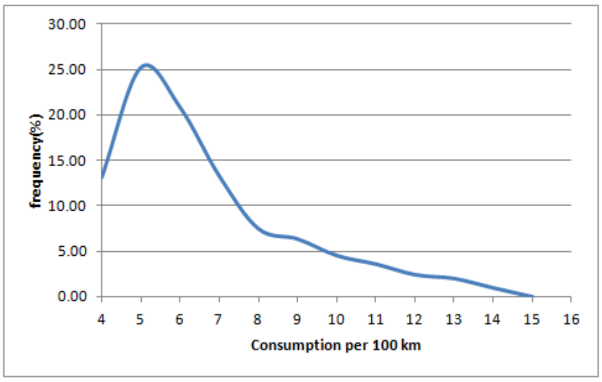

Figure 8. Actual distribution of fuel consumption by cars per 100 km based on the data

As seen in Table 6, the mean of fuel consumption is approximately 6.9 liters per 100 km, and the median is 6.22 liters. This result implies that 50% of the vehicles have less than 6.22 liters consumption per 100 km. Therefore, for considering fuel consumption by a car, the scenarios can be generated using a lognormal distribution with its parameters estimated to fit the data (Evans et al., 2000).

Table 6. Descriptive statistics for the fuel consumption per 100 km by a car

Consumption per 100 km	liter
Mean	6.9
Mode	5.00
Median	6.22
Std. Deviation	2.3

6. Case Study Results

This section presents the results of the case study, first taking into account each stochastic process separately and then analyzing their joint effect on the cost and risk associated with each different type of vehicle.

The generation of the fuel price scenarios for each technology is made with equation (1). As a two-stage problem is considered and decisions are made at the first stage, the average of fuel prices is obtained for each scenario during the planning horizon. Moreover, for the other two stochastic processes, which are mileage driven and fuel consumption, the scenarios are generated based on a fitted distribution, which is a lognormal with the parameters matched with the data, as explained in sections 5.2 and 5.3.

In each set of simulations, when considering each stochastic process separately, a total of 12000 scenarios (4000 scenarios for each of them and 3 states for carbon prices) are used. Moreover, when considering all of the stochastic processes simultaneously, due to the higher complexity of the model, 24000 scenarios (20 scenarios for fuel prices, 20 scenarios for mileage driven, 20 scenarios fuel consumption, and 3 states for carbon prices) are used.

The number of scenarios is obtained based on trial and error for convergence of the model (It has been determined that if the number of scenarios increases, the results will be not be changed. It has also been verified that if they decrease, there will be an inconsistency problem). In addition, the confidence level β equals 0.9. The planning horizon is assumed to be four years from the beginning of 2012 to the beginning of 2016, which is the standard leasing duration of the vehicles.

Regarding the interpretation of the CVaR in the following tables, it should be noted that there is no bad or good CVaR. Indeed, the CVaR itself represents the expected loss faced by the company with a given probability. A way to better interpret the CVaR is to consider the gap between expected cost and CVaR as a measure of risk. The lower this gap, the lower the risk. Moreover, the value of CVaR also depends on the confidence level (β). The closer the value of β is to 1, the higher the values of CVaR: in this case, the fleet manger is more conservative.

First, the impact of fuel prices on the choice of the vehicle to be leased is analyzed. If the expected values for mileage driven presented in Table 5 and fuel consumption depicted in Table 3 are taken into account, then the optimal choice of vehicle, as a function of the weights of the expected cost and of the CVaR, is summarized in Table 7.

In Table 7, when ω changes from 0 to 0.7, i.e., the focus is on minimizing risk rather than minimizing expected cost, the optimal policy is to choose an electrical vehicle from the benchmark brand (b_B). In contrast, by increasing the weight of the expected cost, i.e., ω ranges from 0.7 to 1, the best option is to lease the diesel vehicle, and b_2 is the chosen brand due to its better capital and running costs. Moreover, by choosing a diesel vehicle, there is a reduction in expected cost of approximately K£ 5.91 (25.49%) and an increase in the associated CVaR by K£ 16.06 (59%). That is, by choosing a diesel vehicle, there is an increase in the risk of approximately £ 0.4 per mile for the expected mileage over the planning horizon due to the volatility in fuel prices**.**

Table 7. Results for considering the fuel prices, in 000£, as a stochastic process in the model

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	27.22	26.82	26.02	25.20	24.40	19.88	17.27
expected-cost(K£)	23.18	23.18	23.18	23.18	23.18	17.27	17.27
CVaR(K£)	27.23	27.23	27.23	27.23	27.23	43.29	51.00
VaR(K£)	27.06	27.06	27.06	27.06	27.06	43.29	51.00
Technology Brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

Furthermore, the expected value of the monthly mileage driven by cars, which is 834 miles per month (Table 4), is taken into account. As mentioned in section 5, if this value decreases to approximately 250 miles per month, then rather than a diesel vehicle, a petrol vehicle will be the optimal choice for minimizing the cost. In contrast, if this value increases to approximately 1700 miles per month, then the electric vehicle will be chosen rather than the diesel vehicle (Figure 6). However, for minimizing risk, the electrical vehicle is always the optimal choice regardless of the expected value of the monthly mileage driven in the model (Proposition 1).

Next, the impact of mileage uncertainty on the optimal choice of vehicle is considered. Assuming that the expected values for fuel prices in Table 8 the associated risks (due to fuel price uncertainty) and costs for a vehicle are computed for the planning horizon. The results are depicted in Table 9. When the value of ω is within the range 0 to 0.7 the electric vehicle is the optimal choice. However, for values of ω above 0.7 the diesel vehicle is chosen instead. Moreover, by choosing the diesel vehicle as the optimal choice we have a reduction in total cost which is about K£ 5.84 (25.15%) and increasing the associated CVaR by K£ 42.13 (127.4%). Therefore, leasing an electric car for four years can mitigate the risk due to uncertainty in the mileage driven. A formal proof is provided in Proposition 2.

Table 8. Expected fuel prices (£) from 2012 to the beginning of 2016

	2012	**2013**	**2014**	**2015**	**Average (four years)**
Petrol(£)	1.37	1.40	1.42	1.45	1.41
Diesel(£)	1.41	1.45	1.50	1.55	1.48
Electric(£)	2.54	2.65	2.76	2.87	2.70

Table 9. Results for considering the mileage driven by car, in 000£, as the stochastic process in the model for four years

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	33.06	32.08	30.10	28.13	26.16	23.15	17.36
expected-cost (K£)	23.20	23.20	23.20	23.20	23.20	17.36	17.36
CVaR (K£)	33.07	33.07	33.07	33.07	33.07	75.20	167.00
VaR (K£)	30.16	30.16	30.16	30.16	30.16	58.38	167.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

Next, the impact of fuel consumption in the choice of vehicle is considered. The benchmark fuel consumption for each technology and its brand (Table 3) and the fitted standard deviation for each brand are used to generate the scenarios. The expected values for fuel prices (Table 8) and mileage driven (Table 5) are assumed. The results are summarized in Table 10. As seen by changing the values of ω from 0 to 0.7, it is optimal to choose the electric vehicle for minimizing the risk and cost simultaneously. However, if the value of ω increases more than 0.7 up to 1, then the diesel vehicle is the optimal choice for minimizing cost. Moreover, by choosing the diesel vehicle, there is a reduction in total cost of approximately K£ 6.08 (26.24%) and an increase in the associated CVaR of approximately K£ 29.08 (108.55%). As a result, leasing an electric vehicle significantly decreases the risk due to volatility in fossil fuel consumption. A formal proof for this issue is provided in Proposition 3.

Table 10. Results for considering fuel consumption by petrol and diesel cars, in 000£, as the stochastic process in the model for four years

ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	26.79	26.43	25.71	24.99	24.27	20.97	17.10
expected-cost (K£)	23.18	23.18	23.18	23.18	23.18	17.10	17.10
CVaR (K£)	26.79	26.79	26.79	26.79	26.79	55.87	96.00
VaR (K£)	26.79	26.79	26.79	26.79	26.79	48.38	96.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

Another important issue is the ranking of risk drivers in the model. As seen by comparing Tables 7, 9, and 10, the diesel vehicle (brand b_2) and the electric vehicle (brand b_B) are the optimal choices based on different values of ω. Indeed, if you are more risk averse, you choose electric technology, and if you are more risk neutral, you choose diesel technology with the corresponding brands as the optimal choices. However, the petrol vehicle and brand b_1 and are not competitive with the aforementioned technologies and brands in terms of risk or cost minimization. This is why only the risk drivers of diesel and electric vehicles with associated optimal brands are considered in Figure 9 and 10, respectively.

As seen from Figure 9, the most important risk driver when a diesel vehicle is used with brand b_2 is mileage driven, which has the highest value of CVaR, followed by fuel consumption, and finally by fuel prices. This surprising result is very specific to the data, and it is justified by the large volatility in the distribution of fuel consumption presented in Figure 8.

Furthermore, in Figure 10, the value of CVaR for different risk drivers is represented when EVs of the benchmark brand are used. In this case, the fuel price ranked as the second most important risk factor for EVs in terms of the value of CVaR. In addition, when Figures 9 and 10 are compared in terms of the value of CVaR, as mentioned before in Propositions 1, 2, and 3, the value of CVaR for diesel vehicles is higher than for EVs for each corresponding stochastic process.

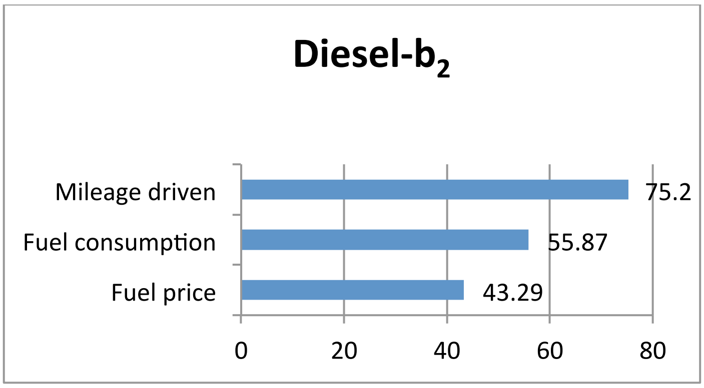

Figure 9. Comparing risk drivers in terms of value of CVaR, in 000£, from 2012 to 2016 for diesel technology with brand b_2.

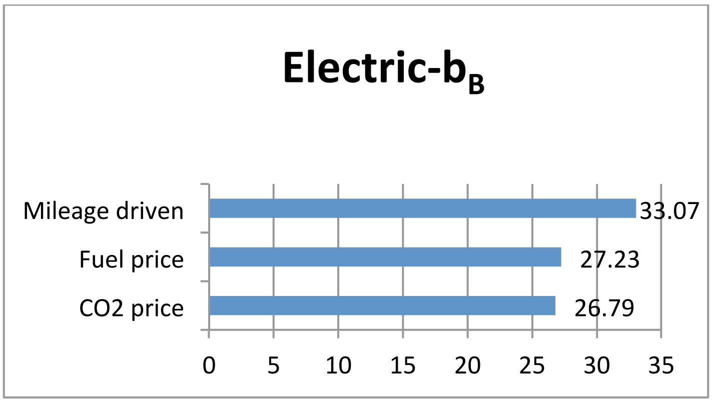

Figure 10. Comparing risk drivers in terms of the value of CVaR, in 000£, for four years from 2012 to 2016 for benchmark brand (b_B).

Now, the complete stochastic model when there is uncertainty due to fuel prices, mileage driven, and fuel consumption is considered. The results for the full model are presented in Table 11. As seen by changing values of ω from 0 to 0.7, the optimal decision is to lease an electric vehicle. However, if the value of ω increases more than 0.7 up to 1, then the diesel vehicle is the best option. Moreover, by choosing the diesel vehicle as the optimal choice, there is a reduction in total cost, which is approximately K£ 6.26 (27.12%), and an increase in the associated CVaR by K£ 34.52 (116.58%). Therefore, as a general conclusion, it seems that leasing an electric vehicle can significantly mitigate risk exposure at an additional expected cost.

One important conclusion, when comparing the CVaR by considering all stochastic processes in the model with the case when only one stochastic process is considered separately is that the CVaR when all of the stochastic processes are considered is less than sum of the CVaRs for the stochastic processes separately. This result is supported by the subadditivity property of coherent measures, as presented in equation (30), Artzner et al. (1997). Therefore, by taking into account inequality (30) and (31), it can be concluded that the analytical results are supported by the computational results in Tables 7, 9, 10, and 11.

$$\phi(X+Y+Z) \le \phi(X)+\phi(Y)+\phi(Z) \tag{30}$$

$$\phi_{ib}^{\beta}(f_{iu}, D_m, o_{bv}) \le \phi_{ib}^{\beta}(f_{iu}) + \phi_{ib}^{\beta}(D_m) + \phi_{ib}^{\beta}(o_{bv}) \tag{31}$$

Lastly, the total cost per mile for each mileage scenario (per month), for the b_2 Diesel vehicle and b_B EV are considered. As seen from Figure 11, the total cost per mile has a decreasing trend as the average mileage driven increases per month in each scenario. Indeed, for high-mileage vehicles, both trends converge to £ 0.34 per mile. However, for normal expected mileage, which is 834 miles per month (Table 5), there is a difference of approximately £ 0.15 per month between the two choices. Therefore, it follows that if the high-mileage case is considered (section 5, Figure 6) and other stochastic processes are included in the decision support model (i.e., fuel prices and fuel consumption), the EV is the optimal choice.

Table 11. Results for considering fuel prices, mileage driven, and fuel consumption, in 000£, as stochastic processes in the model for four years

Ω	**0**	**0.1**	**0.3**	**0.5**	**0.7**	**0.9**	**1**
weighted-cost(K£)	29.61	28.96	27.65	26.35	25.04	21.55	16.82
expected-cost (K£)	23.08	23.08	23.08	23.08	23.08	16.82	16.82
CVaR (K£)	29.61	29.61	29.61	29.61	29.61	64.13	98.00
VaR (K£)	28.96	28.96	28.96	28.96	28.96	55.00	98.00
technology brand	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Electric b_B	Diesel b_2	Diesel b_2

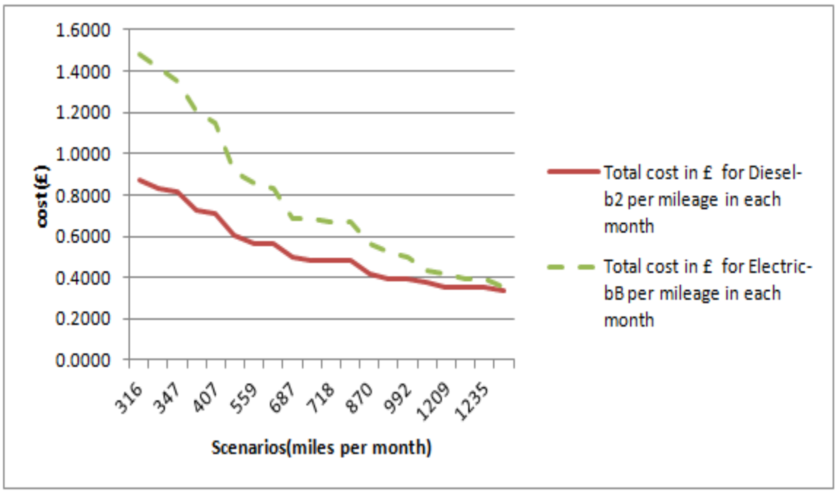

Figure 11. Total cost in £ per mileage for diesel technology with brand b_2 and electric technology with the benchmark brand for each scenario of mileage driven in each month

7. Conclusions

Fleet management is an important tool for reducing CO_2 emissions and fuel costs and improving transportation sustainability. This article proposes a stochastic mixed integer linear programming model that incorporates risk concerns (CVaR) to analyze the choice of technology by a firm that aims to replace some of its vehicles. The firm minimizes expected cost and risk simultaneously, taking into account the uncertainties that exist in the real situation: carbon prices, fuel prices, mileage driven, and fuel consumption.

Specifically, the analytical results show that for each stochastic process of fuel prices, mileage driven, and fuel consumption, the value of CVaR for EVs is less than for fossil fuel vehicles under certain conditions. For example, for the case involving fuel prices treated as a stochastic process, leasing a diesel vehicle rather than an electric vehicle increases the value of CVaR by 59%. This value for mileage driven and fuel consumption is 127.4% and 108.6%, respectively. In addition, the results show that if each stochastic process is considered separately, the most important risk driver for a diesel vehicle is the mileage driven, followed by fuel consumption, and lastly, fuel prices. For the case of EVs, the first important risk factor is mileage, followed by fuel prices and then CO_2 prices.

Furthermore, when all of the stochastic processes are considered together, leasing a diesel vehicle rather than an electric vehicle for four years (2012 to 2016) decreases the total expected cost by approximately K£ 6.26 (27.13%) and increases the associated risk by K£ 34.52 (116.6%) due to uncertainty in the carbon prices, fuel prices, mileage driven, and fuel consumption. Moreover, by considering all stochastic processes together, it can be seen that the risk of the whole model is less than the summation of risk for each stochastic process.

Lastly, by comparing the total cost per mile for each mileage scenario (per month) and including other uncertainty factors in the decision support model, it can be concluded that for high-mileage vehicles, the EV is the optimal choice.

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Online Appendix

As seen from Table A.1, based on the Augmented Dickey-Fuller test, Fuller (1976), the differentiated fuel prices with one order differentiation are stationary because the Null Hypothesis, which suggests that the differentiated fuel price has a unit root, is rejected.

Table A.1. Results for differentiated fuel prices with one order differentiation

Null Hypothesis: D(Petrol) has a unit root		t-statistic	Prob.
Augmented Dickey-Fuller test statistic		-7.86	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	
Null Hypothesis: D(Diesel) has a unit root		t-statistic	Prob.
Augmented Dickey-Fuller test statistic		-7.22	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	
Null Hypothesis: D(Electricity) has a unit root		t-statistic	Prob.
Augmented Dickey-Fuller test statistic		-8.01	0.00
Test critical values:	1% level	-3.47	
	5% level	-2.88	
	10% level	-2.57	

The next step is to compute various criteria to select the lag order of VAR. Table A.2 displays various information criteria for all lags up to the specified maximum. The criterion that has the lowest value between different Lags should be selected. Based on Table A.2, because the Schwarz Information Criterion (SC) and the Akaike information criterion (AIC) (which have similar definitions, Schwarz (1978) and Akaike (1977)) show different lag orders, the third criterion, which is the Hannan-Quinn information criterion (HQ), is also considered. The HQ criterion (Hannan and Barry, 1979) has the lowest value for the lag 1 between different lags; as a result, VAR with lag order equals one is used.

Table A.2. Different values for criteria for choosing the order of Lag

Vector Auto Regression Lag Selection Criteria			
Lag	AIC	SC	HQ
0	-14.56	-14.49	-14.53
1	-14.98	**-14.71**	**-14.87**
2	-14.86	-14.4	-14.67
3	-14.86	-14.2	-14.59
4	-14.93	-14.08	-14.59
5	**-15**	-13.95	-14.57
6	-14.93	-13.68	-14.42
7	-14.88	-13.44	-14.3
8	-14.86	-13.22	-14.19

In the next section, the AR root's graph (Lütkepohl, 1991) is obtained. The estimated VAR is stable (stationary) if all roots have a modulus less than one and lie inside the unit circle. If the VAR is not stable, certain results are not valid. There will be *kp* roots, where *k* is the number of endogenous variables and *p* is the largest lag. Therefore, based on the fact that there are three endogenous variables, which are petrol, diesel, and electricity, and the largest lag order is one (Table A.2), there should be three roots. As can be seen in Figure A.1, all of the roots are inside the unit circle, and the estimated VAR is stable (Lütkepohl, 1991). Lastly, the coefficients for simultaneous equations of VAR are shown in Table A.3.

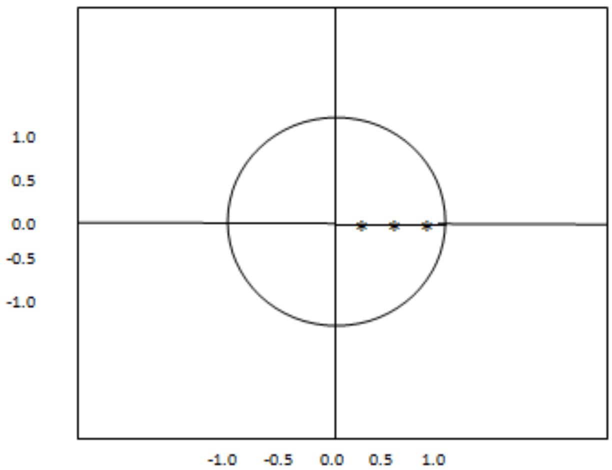

Figure A.1. Unit circle for testing the stability of estimated VAR

Table A.3. The coefficients for solving the VAR model for fuel prices for a sample of data from Jan. 2000 to Dec. 2011

Vector Auto Regression Estimates Standard error in ( ) and t-statistics in [ ]			
	D(Petrol)	D(Diesel)	D(Electricity)
D(Petrol(-1))	0.112	-0.077	0.044
	(0.173)	(0.160)	(0.239)
	[0.646]	[-0.484]	[0.187]
D(Diesel(-1))	0.261	0.506	-0.201
	(0.184)	(0.170)	(0.254)
	[1.41]	[2.96]	[-0.79]
D(Electric(-1))	-0.211	-0.123	0.354
	(0.059)	(0.054)	(0.080)
	[-3.57]	[-2.25]	[4.34]
C	0.004	0.003	0.006
	(0.002)	(0.001)	(0.002)
	[2.09]	[1.81]	[2.11]
R-squared	0.22	0.22	0.143
Log Likelihood	327.46	338.27	282.94
Akaike AIC	-4.654	-4.809	-4.01
Schwarz SC	-4.56	-4.72	-3.92
Mean dependent	0.004	0.004	0.008
S.D. dependent	0.02	0.02	0.03

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