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The impact of platform audits on the manipulation of online reviews.

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The Impact of Platform Audits on the Manipulation of Online Reviews

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ABSTRACT

To reduce sellers' manipulation of online reviews, platforms conduct audits and impose penalties on abusive practices. In this article, we study the effectiveness of self-regulation in preventing fake reviews and increasing consumer and social welfare. To this purpose, we use a Stackelberg model with the platform as the leader. Our major contribution is to explain the auditing game between the platform and the seller. These are our major findings. First, we found a *self-regulation's Achilles' heel*. Despite its noble intent, the self-regulation system often falters. Why? The platform faces a powerful incentive to accommodate fake reviews, especially when consumers heavily rely on the seller's information and regulatory repercussions remain mild. For this reason, consumer activism and external supervision are the key factors leading the platform to audit the seller. Second, we discovered the *paradox of fake reviews*, establishing the conditions under which, counter-intuitively, fake reviews create social value. Finally, by considering an endogenous retail price, we found evidence of a complex relationship between pricing, auditing, and review manipulation policies, all dependent on consumer sophistication and product quality, which can only be analyzed numerically, as there is no closed-form solution.

1. Introduction

Online reviews are an essential source of information about the quality of products and services and can significantly influence consumers' purchase decisions (Fainmesser et al., 2021). Womply Research (2021) reports a strong positive correlation between the number of reviews and firms' sales revenue, as businesses with more than 82 total reviews earn 54% more in annual income than average. Reputation X, an online ethical reputation management services firm, reports that a

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single negative review can drive away 22% of customers, and three negative reviews can drive away a whopping 59% (Bruce, 2020). For this reason, online sellers are incentivized to manipulate online reviews (Cao, 2020). It has been shown that there are many fake reviews on platforms such as Amazon, Yelp, and TripAdvisor (Dellarocas, 2006; Harris, 2018).

Nonetheless, these fake reviews reduce consumers' trust in online review systems and damage the reputation of platforms (Wu et al., 2020). Thus, many platforms have developed anti-manipulation policies. Amazon can suspend or terminate a seller's Amazon privileges, remove reviews, and delist related products if the seller is found to have attempted to manipulate reviews (Amazon, 2021a). In another example, eBay's feedback manipulation policy indicates that the seller has the following risks if he manipulates online reviews: administrative ending or canceling of listings, hiding or demoting all listings from search results, and account suspension (ebay, 2021). Taking a slightly different approach, Taobao has a point accumulation system where the seller will have points deducted if he violates the rules regarding fake transactions. Depending on the degree of manipulation, Taobao may decrease the search ranking of related products, remove the fake reviews, or delist all products (Taobao, 2021).

Even though platforms have policies to combat online review manipulation, some sellers still manipulate reviews. Uberall and The Transparency Company (Uberall, 2021) analyzed four million reviews on the most prominent U.S. local review platforms, finding that Google had the most significant number of questionable reviews (10.7%), followed by Yelp (7.1%), TripAdvisor (5.2%) and Facebook (4.9%). Therefore, platforms conduct audits to verify the authenticity of online reviews and punish sellers if caught manipulating online reviews.

However, previous studies about online review manipulation have neglected the platform's audits (Cao, 2020; Dellarocas, 2006; Zhang et al., 2017). Therefore, it is vital to investigate the game between the platform's auditing procedures and the seller's online review manipulation. Our major contribution is to analyze the platform and seller auditing game, addressing the following research questions. (a) Under which conditions does the platform's self-regulation mechanism improve the quality of consumer information? (b) What are the incentives for the platform to audit the seller and for the seller to manipulate online reviews? (c) What is the role of consumers in determining the quality of online reviews? (d) How does online review manipulation affect consumer and social welfare? (e) How do online review manipulation and platform audits affect the retail price?

To answer these questions, we use a Stackelberg game capturing the interaction between the platform's audit and the seller's manipulation. We consider that a seller sells products on an online platform to consumers making purchase decisions based on online reviews. To attract more purchases, the seller may manipulate reviews. The platform attempts to prevent the manipulation of online reviews by conducting audits. As platforms always announce their anti-manipulation policy in advance, we consider that she is the game's leader. We solve the equilibrium solution of the game and analyze the conditions under which the platform conducts an audit, and the seller manipulates online reviews. Then, we study the impacts of the regulatory penalty, seller's penalty loss, audit cost, and manipulation cost coefficients on the respective optimal decisions and profits. Additionally, we examine the impact of manipulation on consumer and social welfare. Finally, to analyze the effect of online review manipulation and platform audits on the retail price, we extended the main model from exogenous retail price to endogenous retail price.

In this paper, we make the following contributions. First, we analyze the auditing and manipulation game between the platform and the seller and identify a gap in the existing literature. We show that there is a *self-regulation's Achilles' heel*, as the platform's self-regulation system is not always effective. For this reason, consumer activism and external supervision are crucial in motivating the platform to audit the seller. Second, we offer insights for platforms on how to design audit policies to deter online review manipulation. We demonstrate that strict auditing and high penalties can significantly reduce manipulation. Third, we discovered the *paradox of fake reviews*, proving that online review manipulation can positively affect social welfare, as it induces consumers to try products they prefer despite being considered unethical behavior.

The remainder of this paper is organized as follows. We review the relevant literature in Section 2 and discuss the model development in Section 3. In Section 4, we derive the equilibrium of the model, and in Section 5, we explain the main determinants of the optimal policies. We extend the model to include endogenous prices in Section 6. In Section 7 we discuss the deductive process used in this article. Finally, Section 8 summarizes the essential findings and the managerial implications.

2. Literature Review

Our study relates to three streams of research: a) the internal auditing in the supply chain, b) the effects of online product reviews on the firm's operational decisions, and c) the manipulation of online reviews.

2.1. The internal auditing in supply chain management

Our study is related to the literature on core companies auditing their partners to ensure compliance. A common theme in this area is the impact of the major retailer's auditing and penalties on the suppliers' social responsibility.

We review several studies examining the retailer's auditing and penalties effects on the supplier's social responsibility. For example, Plambeck and Taylor (2016) finds that more frequent auditing induces the suppliers to hide information and ignore social responsibility, while Cho et al. (2019) shows that internal audit and high penalty can effectively reduce the use of child labor by the supplier.

We also discuss the audit-penalty mechanisms among multiple retailers and suppliers. Caro et al. (2018) demonstrates that the joint and the shared mechanisms are superior to the independent ones, and Zhang et al. (2022) derives the optimal dynamic auditing policy for the retailer in a supply network.

Finally, we highlight the studies that explore the role of financing and information sharing in influencing the supplier's compliance. Chen et al. (2023a) proposes a mechanism that combines bank financing and buyer reward to improve the supplier's compliance level, and Chen et al. (2023b) analyzes the interaction between the firm's auditing and the platform's information disclosure under uncertain product line strategy.

We also review some studies that address other aspects of the internal audit of the supply chain. For instance, Nikoofal and Gümüş (2020) studies the impact of the retailer's auditing on the supplier's hidden actions when the supplier has private information about the supply risk. They show that the audit benefits both the retailer and the supplier. Li et al. (2023) explores the product quality audit by the C2C platform. They demonstrate that the audit encourages the authentic product seller to use the platform.

In this paper, we extend this literature by analyzing the auditing game between the platform and the seller, who can manipulate online reviews to influence consumer purchases. Specifically, we examine the effects of fake reviews on consumer behavior and the responses of the platform and the retailer.

2.2. The Effects of Online Product Reviews on the Firm's Operational Decisions

We focus the literature review on models studying the impact of online reviews on firms' operational decisions. These are dynamic models in which decisions are taken in sequence. First, early consumers make purchase decisions based on their quality expectations. Then later, consumers decide by using the online reviews from earlier adopters (Huang et al., 2021; Fainmesser et al., 2021; Shin et al., 2023). As there is uncertainty about product quality, and as consumers can learn from the sequential disclosure of reviews, Bayesian learning has been used to model consumer inference of product quality (Guo et al., 2022).

Moreover, some studies focus on the impact of online reviews in a steady state, considering only the number of online reviews when stationary. Many of these articles treat online reviews as additional information for consumers' product quality assessment. They model Consumers' expectation of product quality as the average of idiosyncratic information and online reviews information. In this research stream, Yang et al. (2021) study the impact of online reviews on pricing and profits in a dual-channel supply chain; Qiao and Su (2021) study the effects of online reviews on the closed loop supply chain. Another stream of articles models online reviews in two dimensions: quality attribute and fit attribute; online reviews provide additional information to reduce consumers' uncertainty about product quality and fit (Li et al., 2021; Zhang et al., 2017; Cao, 2020).

2.3. Online Review Manipulation

Sellers can legally adjust pricing (Papanastasiou and Savva, 2017), advertising (Fainmesser et al., 2021), rebates (Yang and Dong, 2018), and strategically withholding or disclosing information about product quality (Guan et al., 2020) to affect online review by influencing early consumers' purchasing experience.

Additionally, some sellers may use non-compliant ways to increase revenue, such as posting positive fake reviews in their own review set or negative fake reviews in competitors' review sets (Cao, 2020). Lee et al. (2018) find that positive manipulation is much more common than negative manipulation. Sellers can give incentives for online reviews through monetary rewards; these reviews tend to be positive (Gutt et al., 2019) and, as reported in Wu et al. (2020), significantly influence consumers' purchase behaviors, merchants' profits, and social welfare.

A few articles have attempted to identify the firm type more likely to manipulate reviews. For example, Hu et al. (2011) analyzes the impact of product attributes on online review manipulation in the publishing industry, concluding that there is a severe problem for non-bestseller books. Similarly, Wu and Qiu (2016) proves that only low-quality sellers fake consumer reviews, and Siering and Janze (2019) find that poorly-graded restaurants have more fake reviews.

Nonetheless, the problem seems more complex, and there are circumstances where the relationship between quality and reviews is not linear. Dellarocas (2006) shows that when the firm's revenue is a convex function of consumers' perceived quality, the high-quality firm is more likely to manipulate reviews. In contrast, when the firm's revenue is a concave function of consumers' perceived quality, the low-quality firm has a higher incentive to manipulate reviews.

Another factor affecting review manipulation by the seller is competition. Zhang et al. (2017) study the degree of seller's manipulation under equilibrium when two competitive sellers sell two products of the same quality but horizontally differentiated. They prove that competing sellers A and B conduct the same degree of manipulation under equilibrium, concluding that online review manipulation competition does not mislead consumers. Cao (2020) also analyzes competition, considering two asymmetrical firms selling a substitute product, examining the impact of manipulation by the inferior firm, the superior firm, and then by both firms simultaneously, proving that a firm may profit from its competitor's manipulation. Moreover, Li et al. (2021) also studies competition by examining how a firm fights against the rival's manipulation strategy, and Wu and Qiu (2023) analyzes the issue of competition intensity, proving that it is not affected by the distortion of online ratings.

Additionally, online review manipulation also affects consumers and reviewers. Chen et al. (2022) analyzes how consumers decide on online reviews by weighing the review-sharing revenue, the moral cost of lying, the review-posting cost, and the rebate from the seller. They proved that review manipulation increases social welfare when consumers' moral and review-posting costs are negligible. Mostagir and Siderius (2023) investigate when the reviewer should retain an excellent reputation to influence consumers' purchasing decisions or accept bribes to misrepresent reviews. They prove that eliminating manipulation can increase the welfare of all market participants.

While the literature on online review manipulation has primarily focused on the impact on consumers and sellers, the platform's role in addressing this issue is crucial, as it serves as the intermediary between these two parties. Chen and Li (2022) demonstrated that seller-manipulated reviews may initially benefit platforms but ultimately harm their revenue. In contrast, Chen et al. (2017) advocated for platforms to incentivize authentic reviews from consumers based on their product experiences. However, fake reviews can mislead consumers, violate consumer protection laws, and erode market efficiency (Hunt, 2015). To address these concerns, platforms have implemented anti-manipulation policies to foster a trustworthy trading environment and curb misleading or deceptive comments. Chen et al. (2019) found that severe penalties can effectively deter sellers from manipulating reviews. In contrast, Chen and Papanastasiou (2021) proposed a moderate-intensity anti-manipulation strategy that effectively leverages manipulation to benefit sellers and consumers.

Table 1
The comparison between our study and closely related literature

References	Online Reviews	Review Manipulation	Audit & Penalty	Game Theory	Welfare
Papanastasiou and Savva (2017)	✓			✓	
Qiao and Su (2021)	✓			✓	✓
Siering and Janze (2019)	✓	✓			
Cao (2020)	✓	✓		✓	
Chen and Papanastasiou (2021)	✓	✓		✓	✓
Fainmesser et al. (2021)	✓	✓		✓	✓
Guo et al. (2022)	✓	✓		✓	✓
Cho et al. (2019)			✓	✓	
Chen et al. (2023b)			✓	✓	
Our study	✓	✓	✓	✓	✓

2.4. Research gap

To provide a clear context for our study within the broader literature, we have compiled a comparative overview of our work and closely related research in Table 1.

While research on online reviews and review manipulation has significantly advanced the field, there has been limited investigation into the actions taken by retail platforms to address fake reviews. Consequently, existing research is less equipped to tackle the auditing manipulation game between platforms and sellers. In reality, manipulation and anti-manipulation often constitute a two-player game between the platform and the seller. The platform weighs the loss due to the regulatory penalty for fake reviews, audit expenses, and revenue shared with sellers. We extend prior research on online reviews and review manipulation by examining a platform that conducts audits and imposes penalties when sellers manipulate reviews. Drawing upon the methodology of supply chain internal audit literature, we delve into the game between a platform's audits and a seller's manipulation strategies. Additionally, we explore the impact of crucial parameters (such as regulatory penalty and consumers' perceived quality) on strategies, profits, and social welfare. In conclusion, exploring the game between platform review processes and sellers' manipulation of online reviews holds significant theoretical and practical value for e-commerce supply chains.

3. The Stackelberg Game

We model the interaction between the platform (leader) and the seller in the manipulation game using the Stackelberg game (e.g., Cho et al., 2019) depicted in Figure 1. In stage 1, the platform sets the audit effort x, catching the manipulative seller with probability x. In stage 2, after observing x, the seller puts the manipulation effort r: the extent to which fake reviews influence consumers' perception of product quality. Then, the platform conducts an audit and removes any fake reviews that are found. In this second stage, if the platform detects the manipulation, the seller loses a proportion ($0 < \eta \leq 1$) of his current revenue; if the seller's manipulation is not detected, consumers' perception of the product quality from online reviews increased r. In stage 3, the consumers make purchase decisions.

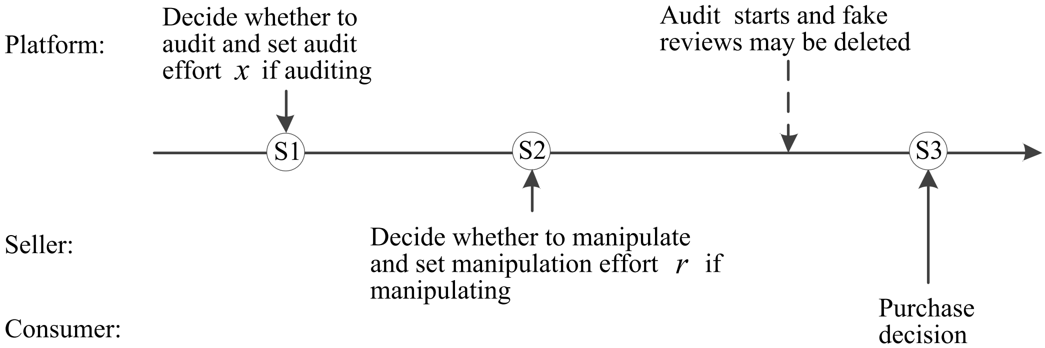


Figure 1: The sequence of the game

We employ backward induction to resolve the Stackelberg game illustrated in Figure 1. We commence by presenting the decision-making processes of consumers (stage 3) in Section 3.1, followed by the seller's choices (stage 2) in Section 3.2, and concluding with the platform's actions (stage 1) in Section 3.3. The fundamental notation utilized throughout this study is summarized in **Appendix A (Table A1)**.

3.1. Consumer Purchase Decisions

We assert that consumer behavior is shaped by the difficulty in perceiving the quality of online products. Because when buying online, consumers cannot try the products and are uncertain about product quality. For this reason, consumers learn about product quality using idiosyncratic information and online reviews. We start by analyzing the modeling of consumer behavior.

3.1.1. The Utility Function.

Without online reviews, consumers form their product quality assessments based on the seller's description, brand, advertisement, and past purchases: this is the consumers' idiosyncratic information, which depends on the individual and differs among consumers (Cao, 2020). For example, when Huawei launches a new phone, such as the HUAWEI Mate 40, compared with people who had never used Huawei handsets before, consumers who have experience with Huawei Mobile and are satisfied with them are more likely to buy the HUAWEI Mate 40. We denote consumers' product quality assessment based on their information source as $\tilde{q}$. This is a uniformly distributed random variable from 0 to 1 (Yang et al., 2021; Yang and Dong, 2018).

Additionally, consumers learn about product quality from online reviews. Let $q \in (0, 1)$ be the quality revealed by unmanipulated online reviews. This quality is assumed to be common knowledge to all consumers. Then, consumers form their product quality assessments based on $\tilde{q}$ and q. Moreover, let β be the weight on idiosyncratic information, $\tilde{q}$, and $1 - \beta$ be the weight on online review information, q when consumers form an assessment of product quality. Then, if there are no fake reviews, the consumers' expected quality of the product is $\beta\tilde{q}+(1-\beta)q$ (Cao, 2020; Yang et al., 2021). A high β means that the online review information is unreliable, and consumers prefer to make purchase decisions based on personal information. The higher the credibility of the review mechanism, the higher $1-\beta$. This β can also capture how product specificity impacts the pattern of online reviews. Insufficient review information for new and niche products makes consumers rely more on idiosyncratic information (high β) when estimating product quality. For mature products, the plenty of online reviews make consumers rely more on online review information (low β). The consumer's expected net utility of the product is equation (1): the difference between the consumers' expected quality of the product and retailer price p.

$$u = \beta\tilde{q} + (1-\beta)q - p \tag{1}$$

We now proceed by describing how fake reviews affect consumer behavior. Let r be the seller's manipulation effort. When fake reviews exist, the product quality revealed by online reviews changes from q to $q + r$.

Although r directly increases the estimate of product quality, it has the downside of reducing trust in online reviews. Consumers may be unable to tell which review is fake, but they know that online reviews are manipulated. Let $\Delta \in [0, 1]$ denote the consumers' *intolerance* of fake reviews. Δ reflects how consumer confidence in online reviews is reduced due to fake reviews in the system. The Δ is such that $0 \leq \beta + \Delta \leq 1$ reflects the characteristics of consumers: the higher the Δ, the less tolerant consumers are to fake reviews. When Δ is high, their trust in online reviews drops significantly when fake reviews exist.

Using these concepts and taking into consideration manipulated online reviews, we can now define the consumers' expected quality of the product as $(\beta+\Delta)\tilde{q}+(1-\beta-\Delta)(q+r)$. Consequently, the consumers' expected net utility is represented by equation (2).

$$u = (\beta + \Delta)\,\tilde{q} + (1 - \beta - \Delta)\,(q + r) - p \tag{2}$$

Cao (2020) makes a similar assumption: consumers have less trust in online review information and rely more on idiosyncratic information when fake reviews exist. In this case, a higher $\beta + \Delta$ means the online review information is less reliable. For this reason, even if the product quality is high (q is close to 1), fake reviews may undermine consumers' assessment of product quality as Δ approaches one.

Although consumers know, in general, whether fake reviews exist in the system (affecting, on average, all the products being sold on the platform), and for this reason, have adjusted their trust level in online reviews, they do not know the actions taken by the platform and the seller about the specific product they want to purchase. There are two ways by which there may be no fake reviews about a particular product: (a) the seller did not manipulate the reviews, or (b) the platform deleted them once the manipulation was detected. Let $\mathbb{I}_r$ denote the seller's manipulation decision: $\mathbb{I}_r = 1$ means that the seller manipulates online reviews, i.e., $r > 0$; and $\mathbb{I}_r = 0$ means that the seller does not manipulate online reviews, i.e., $r = 0$. When the seller manipulates online reviews (i.e., $\mathbb{I}_r = 1$), he fails the audit with probability x and passes the audit with probability $1 - x$. Figure 2 depicts the potential outcome of the game between the platform and the seller: the existence of fake reviews and the matching customer utility.

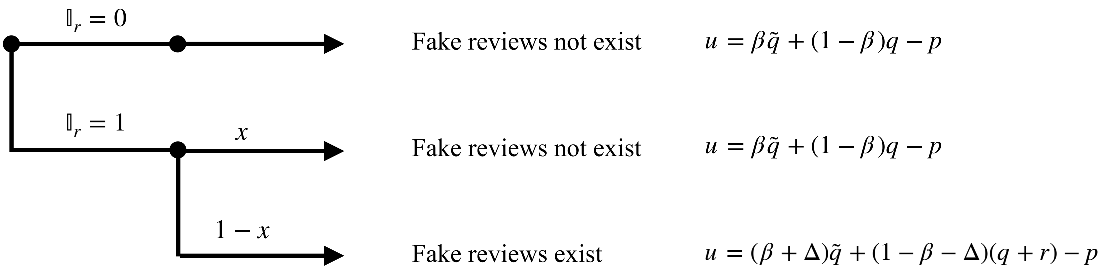


Figure 2: Decision tree

3.1.2. Demand.

We can now proceed by deriving the demand function from the basic properties of consumer behavior.

The consumers buy the product only if their expected net utility is positive. Therefore, by taking the integral $D = \int_{[p-(1-\beta-\Delta)(q+r)]/(\beta+\Delta)}^{1} f(\tilde{q})\, d\tilde{q}$ we derive the demand function $D = 1 - [p - (1 - \beta - \Delta)(q + r)]/(\beta + \Delta)$. If fake reviews do not exist, then $\Delta = 0$ and $r = 0$; however, when fake reviews exist, $\Delta \geq 0$ and $r > 0$.

As depicted in Figure 2, fake reviews depend on both the seller's manipulation and the platform's auditing efforts. Consequently, by combining the demands with the platform's and seller's decisions, we derive the expected demand function, represented by equation (3): when

$\mathbb{I}_r = 1$ the $r > 0$ and the demand $D(r,x) = x\left[1 - \frac{p-(1-\beta)q}{\beta}\right] + (1-x)\left[1 - \frac{p-(1-\beta-\Delta)(q+r)}{\beta+\Delta}\right]$. Moreover, when $\mathbb{I}_r = 0$, then $r = 0$ and the demand $D = 1 - \frac{p-(1-\beta)q}{\beta}$.

$$D(r,x) = x\left[1 - \frac{p-(1-\beta)\,q}{\beta}\right] + (1-x)\left[1 - \frac{p-\left(1-\beta-\mathbb{I}_r\Delta\right)(q+r)}{\beta+\mathbb{I}_r\Delta}\right] \tag{3}$$

3.2. The Seller's Problem

We consider an internet-based retailer using an online platform to sell products. The seller sells a single product for p per unit, and the platform receives a fraction $\alpha \in (0,1)$ of the seller's revenue. Many online retail outlets, e.g., Amazon.com, Tmall.com, and JD.com, have adopted such a fee structure. The platform charges the same agency fee to different sellers (e.g., JD.com sets the same α for all goods in the same category (Ha et al., 2022)), we assume that α is exogenous. Moreover, in the base model, to better explain the game between the platform's audits and seller's review manipulations, we also assume that the price (p) is an exogenous parameter. Several studies have the same assumption, such as Bose and Anand (2007), Caro et al. (2018), and Ma et al. (2021). Later on, in Section 6, we relax this assumption and study the effect of having an endogenous price.

As favorable reviews can increase consumers' expected product quality, the seller may manipulate online reviews by rewarding past consumers for positive reviews to attract more customers (Zhang et al., 2017). For example, sellers usually include a "praise cash-back" card in the package with their products to encourage consumers to post positive reviews. Those who post positive reviews can receive cash or discounts as rebates. As $r \in [0,1]$ is the seller's manipulation effort, he incurs a cost mr^2, with $m \geq 0$. Additionally, the seller suffers a penalty loss ($0 < \eta \leq 1$) as a proportion of current income as a penalty if caught manipulating reviews.

Therefore, the seller earns $p(1-\alpha)\,D(r,x)$ from sales, incurs a manipulation cost mr^2, and suffers an expected revenue loss $x\mathbb{I}_r p(1-\alpha)\eta\{1 - [p-(1-\beta)\,q]/\beta\}$. When the seller uses fake reviews, consumers eventually lose trust in reviews about his products, i.e., the reviews' weight changes from $1-\beta$ to $1-\beta-\Delta$. This change represents a seller's reputation loss from fake reviews.

As a result, after observing x, the seller determines his best response $r(x)$ to maximize his profit (4). In which, when $\mathbb{I}_r = 1$ then $\pi_s(r,x) = p(1-\alpha)D(r,x) - xp(1-\alpha)\eta\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - mr^2$, when $\mathbb{I}_r = 0$ then $\pi_s(r,x) = p(1-\alpha)D(r,x) - mr^2$.

$$\pi_s(r,x) = p(1-\alpha)D(r,x) - x\mathbb{I}_r p(1-\alpha)\eta\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - mr^2 \tag{4}$$

3.3. The Platform's Problem

The platform decides the quality of the auditing process by controlling the probability that the audit will be effective, $x \in [0,1]$, and incurs an audit cost kx^2, where $k > 0$ (Caro et al., 2018; Huang et al., 2022). The more the platform invests in the auditing process, the higher the probability the platform can detect the manipulation when the seller manipulates online reviews.

In addition to self-regulation by the platform, industry regulations also play a significant role in curbing manipulation. When fake reviews are confirmed, the platform is subjected to penalties from the regulatory body. Although external regulation cannot wholly eliminate fake reviews, it catalyzes platforms to conduct audits. Let's denote g as the penalty imposed by external entities. The platform experiences a revenue loss of g when the audit proves ineffective. This assumption aligns with the one made in Cho et al. (2019), where an external inspection of manufacturers auditing suppliers for child labor was considered.

Therefore, the platform earns $p\alpha D(r,x)$ from sales and incurs an audit cost kx^2. Suppose the seller manipulates online reviews (i.e., $\mathbb{I}_r = 1$). In this case, there are two potential outcomes: (a) The seller fails the audit (with probability x), and the platform has an expected revenue loss equal to $x\mathbb{I}_r p\alpha\eta\{1 - [p-(1-\beta)q]/\beta\}$ due to the loss of the seller's income. (b) The seller passes the audit (with probability $1-x$), and the platform suffers an expected revenue loss from the regulatory penalty due to audit failure equal to $\mathbb{I}_r(1-x)g$. At stage 1, the platform anticipates the seller's response and determines x to maximize her expected profit (5): when $\mathbb{I}_r = 1$ the $\pi_p(r,x) =$

$p\alpha D(r,x) - (1-x)g - xp\alpha\eta\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - kx^2$, and when $\mathbb{I}_r = 0$ the $\pi_{\mathrm{p}}(r,x) = p\alpha D(r,x) - kx^2$.

$$\pi_{\mathrm{p}}(r,x) = p\alpha D(r,x) - \mathbb{I}_r(1-x)g - x\mathbb{I}_r p\alpha\eta\left[1 - \frac{p-(1-\beta)\,q}{\beta}\right] - kx^2 \tag{5}$$

4. The Equilibrium of the Stackelberg Game

We start by analyzing consumer behavior as the equilibrium is derived by backward induction. By comparing the demand with and without manipulation, i.e., $D(r,x\,|\,\mathbb{I}_r = 1)$ and $D(r,x\,|\,\mathbb{I}_r = 0)$, we derive Lemma 1: when the consumers' expected net utility solely arising from unmanipulated reviews is negative (i.e., $q < p$) manipulation leads to high demand, i.e. $D(r,x\,|\,\mathbb{I}_r = 1) > D(r,x\,|\,\mathbb{I}_r = 0)$, because when $q < p$ the product will not sell unless the reviews are manipulated.

Lemma 1. *(1) If $q < p$ then $D(r,x\,|\,\mathbb{I}_r = 1) > D(r,x\,|\,\mathbb{I}_r = 0)$; (2) If $q \geq p$ then $D(r,x\,|\,\mathbb{I}_r = 1) > D(r,x\,|\,\mathbb{I}_r = 0)$ when $r > \frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$, and $D(r,x\,|\,\mathbb{I}_r = 1) \leq D(r,x\,|\,\mathbb{I}_r = 0)$ when $r \leq \frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$. (All proofs are included in the Appendix B.)*

On the other hand, when the consumers' expected net utility solely arising from unmanipulated reviews is non-negative (i.e., $q \geq p$): a) manipulation leads to a high demand when the manipulation effort r is large, i.e., $r > \frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$; b) demand decreases due to manipulation when r is small, i.e., $r \leq \frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$.

Therefore, the consumer's intolerance of fake reviews is critical to whether the manipulation affects consumer entry and exit decisions. *When the consumers' intolerance to fake reviews is low, i.e., Δ approximates zero, a small manipulation effort leads to higher sales.* However, if the consumers' intolerance to fake reviews is high, Δ approaches 1, and β converges to zero, online review manipulation decreases demand.

Next, by evaluating the impact of manipulating online reviews on the seller's expected profit, we obtain the result summarized in Lemma 2, in which $r(x)$ is the seller's best response.

Lemma 2. *There is a threshold x_1, described in **Appendix A (Table A2)**, such that if $0 \leq x \leq x_1$ then $r(x) = p(1-\alpha)(1-x)(1-\beta-\Delta)/(2m(\beta+\Delta))$; otherwise if $x_1 < x \leq 1$ then $r(x) = 0$.*

Lemma 2 indicates that the *audit effort x is the primary constraint for the seller's manipulation decision.* There is a bound on the auditing effort, x_1, above which the seller no longer manipulates online reviews. Next, by analyzing the impact of manipulation cost and demand parameters on the seller's manipulation decision, we derive Corollary 1.

Corollary 1. *Let $q > p$. There is a threshold m_1, described in Appendix A (Table A2), such that if $m > m_1$ the seller's best response is $r(x) = 0$, for all x.*

Corollary 1 shows that *when consumers' expected net utility solely arising from unmanipulated reviews is positive (i.e., $q > p$) and the manipulation cost coefficient m is large (i.e., $m > m_1$), the seller does not manipulate online reviews regardless of audit effort.* This means that in addition to the platform's audit effort, product pricing, online ratings, and manipulation cost all play an essential role in sellers' manipulation decisions.

Finally, we analyze the properties of the platform's decision. By substituting Lemma 2 into equation (5), we derive the platform's optimal audit probability and the seller's optimal manipulation effort, as summarized in Proposition 1. The analysis shown in Proposition 1 is innovative because it studies how the regulatory penalty, audit, and manipulation costs impact the auditing and manipulation strategies. It also identifies under what conditions the platform does not audit, the seller does not manipulate, and when they set high, medium, or no audit and low manipulation efforts.

Proposition 1. *The platform's optimal audit probability and the seller's optimal manipulation effort are described in Table 2.*

From Proposition 1, let us further consider the particular case in which the platform does not conduct an audit (i.e., $x = 0$), summarized in Corollary 2.

Corollary 2. *When the platform does not perform audits (i.e., $x = 0$): (1) Only when $q < p + \frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4m(\beta+\Delta)\Delta}$ does the seller manipulate online reviews, and $r^* = \frac{(1-\alpha)(1-\beta-\Delta)p}{2(\beta+\Delta)m}$; (2) Otherwise, the seller does not manipulate online reviews even when the platform does not require an audit.*

Corollary 2 proves that the seller does not manipulate online reviews when the consumers' perceived quality revealed by unmanipulated reviews (q) is sufficiently large, even if the platform does not audit. In this case, as q is higher than the required threshold, online word of mouth is so

Table 2
Equilibrium Solution.

	$g \geq g_3$	$0 \leq g < g_3$
$k \geq k_1,\ m \leq m_0$ $k < k_1,\ m \leq m_1$	(2) high x^* low r^* $x^* = x_1,\ r^* = \frac{(1-\alpha)(1-x_1)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$	(4) $x^* = 0$ high r^* $x^* = 0,\ r^* = \frac{(1-\alpha)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$

	$g \geq g_2$	$g_1 \leq g < g_2$	$0 \leq g < g_1$
$k \geq k_1$ $m_0 < m < m_1$	(2) high x^* low r^* $x^* = x_1$ $r^* = \frac{(1-\alpha)(1-x_1)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$	(3) medium x^* and r^* $x^* = \frac{\beta(\beta+\Delta)^2 gm - \alpha pA + mp\alpha(\beta+\Delta)B}{2km\beta(\beta+\Delta)^2 - \alpha Ap}$ $r^* = \frac{[(2k-g)\beta(\beta+\Delta) - p\alpha B]A}{2\beta(1-\beta-\Delta)\left(2km\beta(\beta+\Delta)^2 - \alpha Ap\right)}$	(4) $x^* = 0$ high r^* $x^* = 0$ $r^* = \frac{(1-\alpha)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$
$q > p,\ m \geq m_1$	(1) $x^* = 0,\ r^* = 0$		
Notes	$A = p\beta(1-\alpha)(1-\beta-\Delta)^2$	$B = (p-q)\left(\Delta + (\beta+\Delta)\eta\right) - (1-q)\,\beta(\beta+\Delta)\eta$	

Table 3
The expected profits of the platform and the seller

	$g \geq g_3$	$0 \leq g < g_3$
$k \geq k_1,\ m \leq m_0$ $k < k_1,\ m \leq m_1$	(2) high x^* low r^* $\pi_s{}^* = \frac{(1-\alpha)pI_1}{\beta},\ \pi_p{}^* = \frac{\alpha pI_1}{\beta} + \frac{\alpha Ap(1-x_1)^2 - 4\beta(\beta+\Delta)^2 mI_3}{4\beta(\beta+\Delta)^2 m}$	(4) $x^* = 0$ high r^* $\pi_s{}^* = \frac{(1-\alpha)p(A+4\beta(\beta+\Delta)mI_2)}{4\beta(\beta+\Delta)^2 m},\ \pi_p{}^* = \frac{\alpha p(A+2\beta(\beta+\Delta)mI_2)}{2\beta(\beta+\Delta)^2 m} - g$

	$g \geq g_2$	$g_1 \leq g < g_2$	$0 \leq g < g_1$
$k \geq k_1$ $m_0 < m < m_1$	(2) high x^* low r^* $\pi_s{}^* = \frac{(1-\alpha)pI_1}{\beta},\ \pi_p{}^* = \frac{\alpha pI_1}{\beta}$ $+\frac{\alpha Ap(1-x_1)^2 - 4\beta(\beta+\Delta)^2 mI_3}{4\beta(\beta+\Delta)^2 m}$	(3) medium x^* and r^* $\pi_s{}^* = \frac{I_6^2 A^2 m}{4I_5^2\beta^2(1-\beta+\Delta)^2} + \frac{I_4(1-\alpha)Bp}{I_5\beta(\beta+\Delta)} + \frac{I_2(1-\alpha)p}{(\beta+\Delta)}$ $\pi_p{}^* = \frac{\left(I_4 - \alpha Ap\right)(\alpha Bp + \beta(\beta+\Delta)g)}{2I_5\beta(\beta+\Delta)} + \frac{\alpha Akp}{I_5} + \frac{I_2\alpha p}{(\beta+\Delta)} - g$	(4) $x^* = 0$ high r^* $\pi_s{}^* = \frac{(1-\alpha)p(A+4\beta(\beta+\Delta)mI_2)}{4\beta(\beta+\Delta)^2 m}$ $\pi_p{}^* = \frac{\alpha p(A+2\beta(\beta+\Delta)mI_2)}{2\beta(\beta+\Delta)^2 m} - g$
$q > p,\ m \geq m_1$	(1) $\pi_s{}^* = \frac{p(1-\alpha)I_1}{\beta},\ \pi_p{}^* = \frac{p\alpha I_1}{\beta}$		
Notes	$I_1 = q - p + \beta(1-q)$ $I_4 = \alpha(\beta+\Delta)Bmp + \beta(\beta+\Delta)^2 gm - \alpha Ap$	$I_2 = q - p + (\beta+\Delta)(1-q)$ $I_5 = 2\beta(\beta+\Delta)^2 km - \alpha Ap$	$I_3 = kx_1^2 + g(1-x_1)$ $I_6 = \beta(\beta+\Delta)(2k-g) - \alpha Bp$

positive that the seller does not need to manipulate online reviews to increase sales, avoiding the risk of reducing the consumer's trust in the reviews.

After deriving the optimal audit and manipulation efforts, we replace x^* and r^* into equations (4) and (5). The derived analytical expressions of the optimal profits, both for the platform and the seller, are summarized in Table 3. These expressions allow us to understand better the impact of the regulatory penalty and audit and manipulation costs on profits.

5. Identifying the Major Determinants of the Optimal Policies

This section analyzes the properties of the Stackelberg equilibrium, describing how the equilibrium outcomes regions changed with cost and demand parameters in Section 5.1, how cost parameters affect the equilibrium solution in Section 5.2, and how fake reviews affect consumer welfare in Section 5.3.

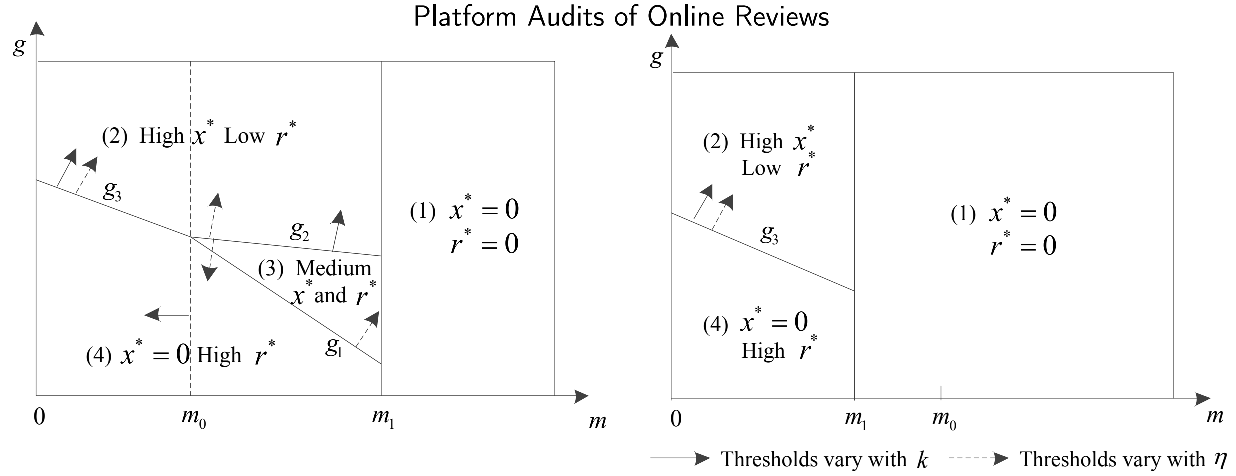
5.1. Impact of Parameters on the Equilibrium Policies

We start by splitting the equilibrium outcomes in Table 2 into four regions by manipulation cost coefficient (m) and regulatory penalty (g), as in Cho et al. (2019). The resulting Figure 3 illustrates how the regulatory penalty (g) and manipulation cost coefficient (m) affect the optimal policies. As g increases, the platform increases audit effort if $m < m_1$; if $m > m_1$, the platform does not audit. Similarly, as g increases and $m < m_1$, the seller decreases the manipulation effort, and if $m > m_1$, the seller does not manipulate. Figure 3 shows that when the manipulation cost is sufficiently large ($m > m_1$), the platform does not audit because the seller does not manipulate (due to the high cost of doing so).

Additionally, Figure 3 also shows that the platform only audits reviews when the regulatory penalty is above some minimum threshold (i.e., $g > min\{g_1, g_3\}$), which is higher the lower the manipulation cost coefficient (m): when manipulation is cheaper (more expensive) the regulatory penalty required to prompt auditing is higher (lower).

Next, we analyze how the costs and demand parameters affect the regions in Figure 3. The first is the audit cost coefficient k. The solid arrows in Figure 3 depict the influence of k on threshold lines: when the audit cost coefficient (k) increases, g_2 and g_3 move up and m_0 moves to the left. Consequently, the platform prefers no or medium audit effort. As a result, the increase in audit costs reduces the incentive to audit.

The second is the seller's penalty loss η. The dashed arrows in Figure 3 depict the influence of the seller's penalty loss (η) on the threshold lines. When η increases, g_1 and g_3 increase, whereas g_2 may increase or decrease. Thus, the platform prefers no audit when the seller's penalty loss is



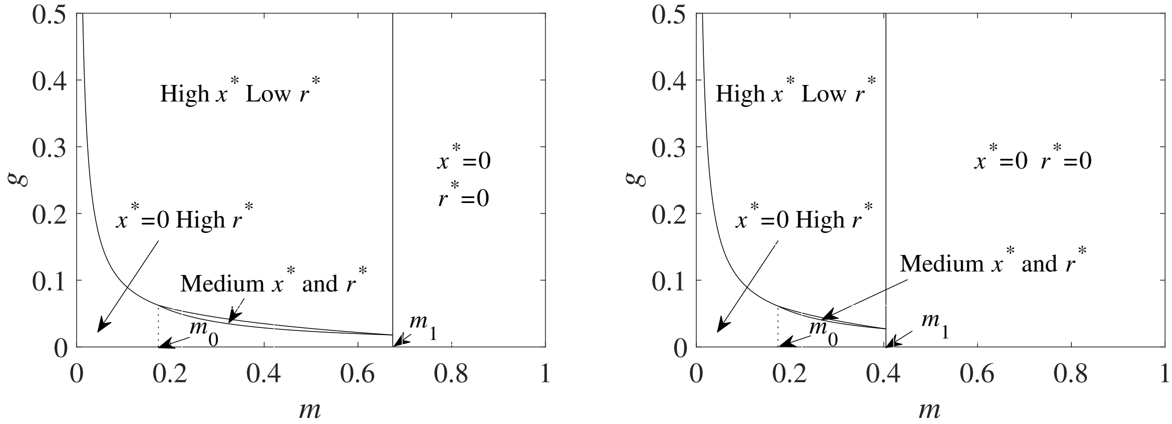
(a) $k \geq k_1$ (b) $k < k_1$

Figure 3: Equilibrium outcomes change with k and η.

high because her revenue comes from the seller's sales. Hence, *the higher penalty loss for the seller reduces the platform's income, thus decreasing the platform's incentive to audit: a shocking failure of the self-auditing mechanism.*

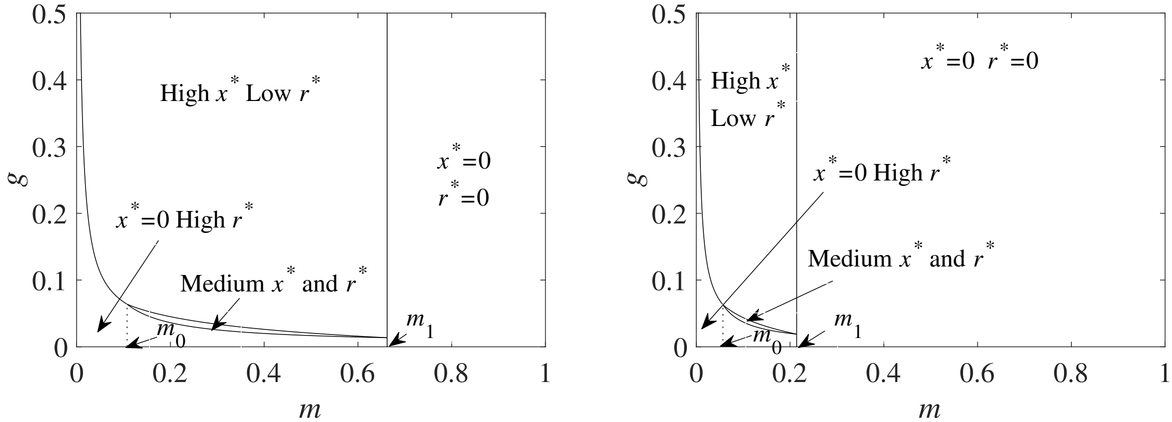
The third and the fourth are the consumer's perceived quality revealed by unmanipulated online review information q and the price p. As the influence of q and p on the equilibrium thresholds g_1, g_2, and g_3 is complex, we provide numerical examples to illustrate how they affect the equilibrium. Following the Selling on Amazon Fee Schedule (Amazon, 2021b), we set $\alpha = 0.08$ and, additionally, we assume that consumers have the same trust in idiosyncratic information as in online reviews (i.e., $\beta = 0.5$). The other parameters are $\Delta = 0.15$, $p = 0.7$, $q = 0.67$, $\eta = 0.25$, $k = 0.03$. Figures 4 and 5 summarize the optimal solutions as q and p change from 0.85 to 0.95 and from 0.55 to 0.4, respectively.

Figure 4 shows that when the consumers' perceived quality revealed by unmanipulated reviews (q) increases, m_1 moves left (which can be verified by $\partial m_1/\partial q < 0$). Consequently, the platform and seller prefer no audit and no manipulation. *A high q is synonymous with excellent online word-of-mouth; sellers do not need to manipulate online reviews.* For this reason, the platform does not conduct audits.



(a) $q = 0.85$, $p = 0.7$ (b) $q = 0.95$, $p = 0.7$

Figure 4: The impact of q on the equilibrium thresholds.



(a) $p = 0.55$, $q = 0.67$ (b) $p = 0.4$, $q = 0.67$

Figure 5: The impact of p on the equilibrium thresholds.

Figure 5 shows that when the selling price (p) decreases, m_0 and m_1 move left (which can be verified by $\partial m_0/\partial p > 0$ and $\partial m_1/\partial p > 0$). Consequently, the platform and the seller prefer no audit and no manipulation, respectively. Figure 5 validates the following conclusion reached by analyzing Figure 3: *the seller does not manipulate reviews of low-price products*.

5.2. Analysis of the Determinants of Profitability

In this section, we examine the effects of the parameters on the optimal policies and profits, as summarized in Proposition 2 and Propositions 3, in which the symbol "+" represents a positive

Table 4
Sensitivity analysis concerning g, η, k and m

Parameters	g				η				k				m			
Solutions	x	r	π_s	π_p	x	r	π_s	π_p	x	r	π_s	π_p	x	r	π_s	π_p
(2) high x^* low r^*				−	−	+		−				−	−	−		−
(3) medium x^* and r^*	+	−	−	−	−	+	∪	−	−	+	+	−	+	−	−	−
(4) $x^* = 0$ high r^*				−										−	−	−

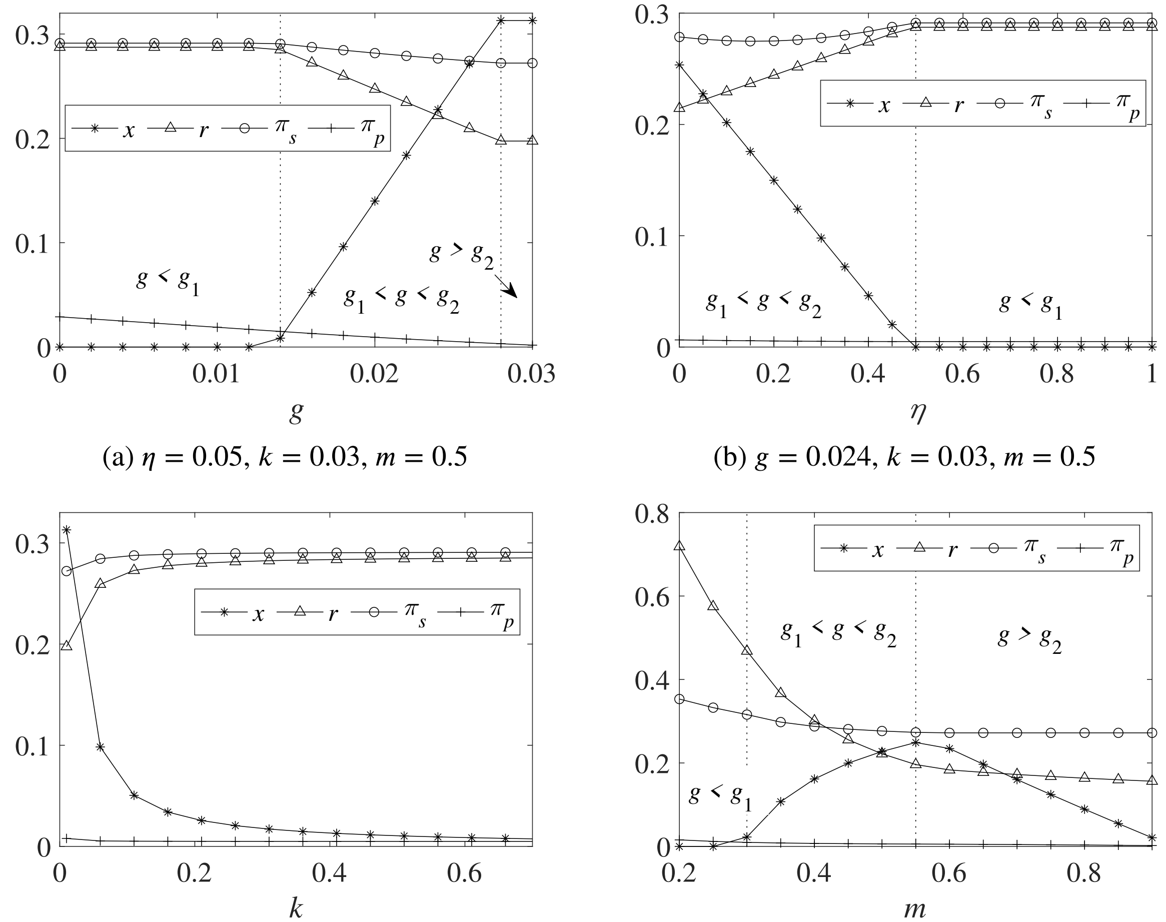
influence, "−" represents a negative influence, a "space" indicates no relationship, and "∪" signals a convex relationship between the parameter and the respective variable.

Proposition 2. *The influence of g, η, k and m on x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$ are described in Table 4. (Note: Case (1), when $x^* = 0$ and $r^* = 0$, is not included in this table as the parameters do not impact the variables.)*

Furthermore, to better explain the results in Proposition 2 , we illustrate the influences of g, η, k, and m on x^*, r^*, ${\pi_p}^*$, and ${\pi_s}^*$ in Figure 6. The parameters are $\alpha = 0.08$, $\beta = 0.5$, $\Delta = 0.15$. To better illustrate the trend, we set $p = 0.58$ ($p < q$) and $m_0 < m < m_1$.

Figure 6(a) illustrates the impact of g on key variables: x^* (audit effort), r^* (manipulation effort), ${\pi_p}^*$ (platform profit), and ${\pi_s}^*$ (seller profit). When the platform faces a substantial regulatory penalty (indicated by a large g), it intensifies audit efforts (x^*) to mitigate revenue losses from penalties. Simultaneously, the seller reduces manipulation efforts (r^*) due to the heightened risk of detection. Consequently, the seller's sales decrease, reducing profits for both the platform and the seller. For this reason, the regulators play a crucial role by imposing penalties when manipulation occurs. When there is regulatory failure to penalize the platform for the presence of fake reviews in the system, the optimal policy is for the platform to accommodate the seller's manipulation of online reviews. *This is the Achilles' heel of self-regulation.*

Figure 6(b) illustrates the influence of η on x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$. From Figure 6(b), counter-intuitively, when the η increases, the platform reduces audit effort (x^*) to zero. This is because the platform's profit comes from the seller's revenue. The platform suffers an expected loss if the seller has a significant penalty loss due to failed audits (a large η). But, from the seller's perspective, as the probability of being caught is zero, and it is still slightly profitable to manipulate, he increases



(a) $\eta = 0.05$, $k = 0.03$, $m = 0.5$

(b) $g = 0.024$, $k = 0.03$, $m = 0.5$

(c) $g = 0.024$, $\eta = 0.05$, $m = 0.5$

(d) $g = 0.024$, $\eta = 0.05$, $k = 0.03$

Figure 6: Sensitivity analysis concerning g, η, k, and m

his effort (r^*). For example, in the case of Taobao, if the platform finds the seller manipulating reviews, Taobao may decrease the search ranking of the product or delist the product. As a result, the seller loses part of the sales revenue, and the platform loses her transaction fees; this is the loss-sharing effect (Geng et al., 2018). This conclusion validates the results shown in Figure 3: the higher η, the higher the optimal r^* chosen by the seller. Another unexpected result is the seller profit: as η rises, the seller's profit follows a U-shaped curve rather than dropping continuously, while the platform's profit declines. This is because the platform is also affected by the seller's loss. Consequently, the platform reduces the audit effort, increasing seller manipulation. *For this* *reason, when the platform's revenue is solely based on the sellers' sales, she does not punish the seller because it would reduce her revenue.*

Figure 6(c) illustrates the influence of k on x^*, r^*, $\pi_p{}^*$ and $\pi_s{}^*$: *when the platform's audit cost coefficient (k) increases, she reduces the audit effort (x^*) and the seller raises the manipulation effort (r^*).* The seller's profit increases because of the increase in manipulation effort (r^*) and the decrease in audit effort (x^*). Additionally, decreasing the audit effort increases the platform's reputation loss (which is not compensated by the increased revenue due to the extra commissions paid by the seller). Thus, the platform's profit decreases.

Figure 6 (d) illustrates the influence of m on x^*, r^*, $\pi_p{}^*$ and $\pi_s{}^*$. When the seller's manipulation cost coefficient (m) increases, he reduces the manipulation effort (r^*), as expected. A quadratic relationship exists between m and the audit effort x^*. When m is sufficiently low in Figure 6 (d), the platform does not audit as the regulatory penalty g is smaller than g_1. When m increases to the mid-range, the platform raises the audit effort (x^*) to restrain manipulation. The platform reduces the audit effort when m consistently increases to a sufficiently large value (x^*). This is because the seller lowers the manipulation effort due to the higher manipulation cost (r^*). Thus, there is no need to audit frequently. Consequently, both the platforms' and the sellers' profits decrease due to increased manipulation cost. The non-linear relationship between the manipulation cost and the audit effort is unexpected: *the platform does not audit more when the manipulation cost is low.*

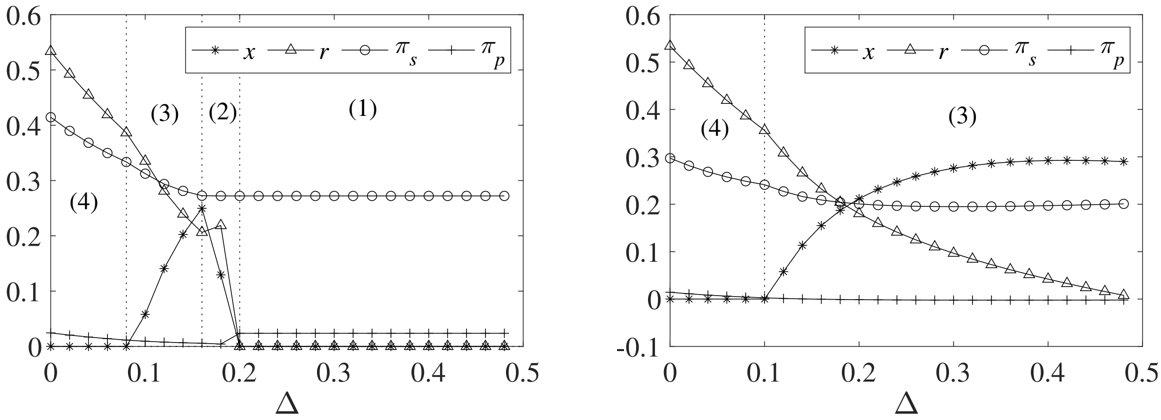
Next, Proposition 3 summarizes the collective impact of Δ and g on x^* and r^*. To avoid the distortion by complicated mathematics, we only consider the interior point solution, i.e., medium x^* and r^*.

Proposition 3. *In equilibrium, there are threshold values $g_x \leq g_r$ such that:*
(a) if $g < g_x$, then $\frac{\partial x^*}{\partial \Delta} > 0$, $\frac{\partial r^*}{\partial \Delta} < 0$;
(b) if $g_x \leq g \leq g_r$ then $\frac{\partial x^*}{\partial \Delta} \leq 0$, $\frac{\partial r^*}{\partial \Delta} \leq 0$;
(c) if $g > g_r$, then $\frac{\partial x^*}{\partial \Delta} < 0$, $\frac{\partial r^*}{\partial \Delta} > 0$.

Proposition 3 shows that the impact of the consumer's intolerance to fake reviews (Δ) on optimal policies hinges on the regulatory penalties (g). When the platform faces mild regulatory consequences, it increases the audit effort when consumers' intolerance for fake reviews increases. For this reason, the seller reduces the manipulation effort. In the second case, when the platform faces medium regulatory penalties, with the increase in consumers' intolerance for fake reviews increases, the platform decreases the audit effort, consequently, the seller decreases the manipulation effort. In the third case, the auditing is initially very high and the manipulation is very low (due to the high penalties). With the increase in consumers' intolerance for fake reviews the auditing effort decreases and, for this reason, the manipulation increases.

Corollary 3. *If the platform does not conduct an audit (i.e., $x^* = 0$), the higher the Δ, the lower the r^*.*

Corollary 3 indicates that if the platform does not conduct an audit and the seller manipulates. Then, as expected, the seller reduces the manipulation effort when the consumer's intolerance for fake reviews increases.



(a) $q > p$ ($q = 0.67$, $p = 0.58$)

(b) $q < p$ ($q = 0.45$, $p = 0.58$)

Figure 7: Impact of Δ on the optimal policy and profitability.

Furthermore, to better explain Proposition 3 and Corollary 3, we illustrate the influences of Δ and q on x^*, r^*, π_p^* and π_s^* in Figure 7. The parameters are the same as in Figure 6. From Figure 7 (a) it is evident that when $q > p$, as Δ increases the platform and the seller change strategy from ($x^* = 0$, high r^*) to (medium x^* and r^*), to (high x^*, low r^*), and then to ($x^* = 0$, $r^* = 0$). From Figure 7 (b), we can observe that when $q < p$, the platform and the seller change their strategy from ($x^* = 0$, high r^*) to (medium x^* and r^*). Therefore, when consumers are less intolerant of fake reviews, the platform does not audit, and the seller uses fake reviews more often. In contrast, the seller reduces manipulation efforts when consumers have a higher intolerance for fake reviews. *Consumer intolerance of fake reviews is an effective way to minimize review manipulation.* Moreover, both the platform and the seller have a lower profit when consumers are less tolerant of fake reviews.

5.3. The Paradoxical Impact of Fake Reviews on Consumer Welfare

We start by formulating consumer welfare. It is the total benefit of consumers derived from buying the product (Zhang et al., 2017; Cohen et al., 2022), can be formulated by equation (6). In equation (6), if the seller manipulates online reviews, i.e., $\mathbb{I}_r = 1$, consumer welfare is $\int_{\frac{p-(1-\beta)(q+r)}{\beta}}^{1} [\beta\tilde{q} + (1-\beta)\, q - p]\, d\tilde{q}$ with probability x and $\int_{\frac{p-(1-\beta-\Delta)(q+r)}{(\beta+\Delta)}}^{1} [(\beta+\Delta)\tilde{q} + (1-\beta-\Delta)\, q - p]\, d\tilde{q}$ with probability $1-x$. If the seller does not manipulate, i.e., $\mathbb{I}_r = 0$, then consumer welfare is $\int_{\frac{p-(1-\beta)(q+r)}{\beta}}^{1} [\beta\tilde{q} + (1-\beta)\, q - p]\, d\tilde{q}$ with probability 1.

$$
\begin{aligned}
w_c &= x \int_{\frac{p-(1-\beta)q}{\beta}}^{1} [\beta\tilde{q} + (1-\beta)\, q - p]\, d\tilde{q} + (1-x) \int_{\frac{p-(1-\beta-\mathbb{I}_r\Delta)(q+r)}{(\beta+\mathbb{I}_r\Delta)}}^{1} \left[(\beta+\mathbb{I}_r\Delta)\tilde{q} + \left(1-\beta-\mathbb{I}_r\Delta\right) q - p\right] d\tilde{q} \\
&= \frac{x[\beta+(1-\beta)q-p]^2}{2\beta} + \frac{(1-x)\left[\left(1-\beta-\mathbb{I}_r\Delta\right)(q-r)+\beta+\mathbb{I}_r\Delta-p\right]\left[\left(1-\beta-\mathbb{I}_r\Delta\right)(q+r)+\beta+\mathbb{I}_r\Delta-p\right]}{2(\beta+\mathbb{I}_r\Delta)}
\end{aligned} \tag{6}
$$

It follows from (6) that when there are no fake reviews, consumer welfare equals to $\frac{[\beta+(1-\beta)q-p]^2}{2\beta}$; whereas when fake reviews exist, consumer welfare equals to $\frac{[(1-\beta-\Delta)(q-r)+\beta+\Delta-p][(1-\beta-\Delta)(q+r)+\beta+\Delta-p]}{2(\beta+\Delta)}$. We obtain Proposition 4 by comparing consumer welfare with and without manipulation.

Proposition 4 (The Paradox of Fake Reviews). *The manipulation increases consumer welfare when* $r \leq \sqrt{\frac{\Delta\left[\beta(\beta+\Delta)(1-q)^2-(q-p)^2\right]}{\beta(1-\beta-\Delta)^2}}$.

Proposition 4 describes the Paradox of Fake Reviews in which a little manipulation makes consumers overestimate product quality, increasing sales and decreasing consumer marginal utility by a smaller proportion. Consequently, as the proportion of increase in sales is higher than the proportion of decrease in marginal utility, consumer welfare increases, leading to the Paradox of Fake Reviews.

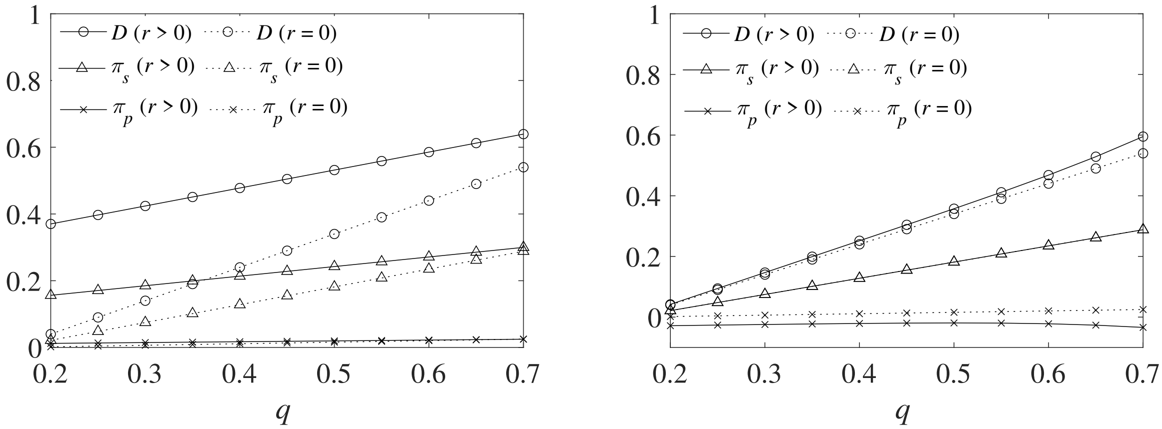
Table 5
Consumers' welfare and social welfare

<table>
<tr><td></td><td>$g \geq g_3$</td><td colspan="2">$0 \leq g < g_3$</td></tr>
<tr><td>$k \geq k_1$
$m \leq m_0$</td><td rowspan="2">(2) high x^* low r^*
$w_c{}^* = \frac{I_1^2 x_1}{2\beta} + \frac{I_9}{8\beta^2(\beta+\Delta)^3 m^2}$
$w_s{}^* = \frac{I_7 x_1}{2\beta} + \frac{2I_{11}\beta(\beta+\Delta)(1-x_1)^2+I_9}{8\beta^2(\beta+\Delta)^3 m^2} + \frac{I_8(1-x_1)-kx_1^2(\beta+\Delta)}{(\beta+\Delta)}$</td><td colspan="2" rowspan="2">(4) $x^* = 0$ high r^*
$w_c{}^* = \frac{4I_2^2(\beta+\Delta)^2\beta^2 m^2 - A^2}{8(\beta+\Delta)^3\beta^2 m^2}$
$w_s{}^* = \frac{I_2^2+2I_8}{2(\beta+\Delta)} + \frac{2I_{11}\beta(\beta+\Delta)-A^2}{8\beta^2(\beta+\Delta)^3 m^2}$</td></tr>
<tr><td>$k < k_1$
$m \leq m_1$</td></tr>
<tr><td></td><td>$g \geq g_2$</td><td>$g_1 \leq g < g_2$</td><td>$0 \leq g < g_1$</td></tr>
<tr><td>$k \geq k_1$
$m_0 < m < m_1$</td><td>(2) high x^* low r^*
$w_c{}^* = \frac{I_1^2 x_1}{2\beta} + \frac{I_9}{8\beta^2(\beta+\Delta)^3 m^2}$, $w_s{}^* = \frac{I_7 x_1}{2\beta}$
$+ \frac{2I_{11}\beta(\beta+\Delta)(1-x_1)^2+I_9}{8\beta^2(\beta+\Delta)^3 m^2} + \frac{I_8(1-x_1)-kx_1^2(\beta+\Delta)}{(\beta+\Delta)}$</td><td>(3) medium x^* and r^*
$w_c{}^* = \frac{I_1^2 I_4}{2I_5\beta} + \frac{I_6 m I_{10}}{8I_5^3\beta^2}$, $w_s{}^* = \frac{I_6^2 I_{11}}{4I_5^2\beta}$
$-\frac{I_4^2 k}{I_5^2} + \frac{I_6 m I_{10}}{8I_5^3\beta^2} + \frac{I_6 m I_8}{I_5} + \frac{I_4 I_7}{2I_5\beta}$</td><td>(4) $x^* = 0$ high r^*
$w_c{}^* = \frac{4I_2^2(\beta+\Delta)^2\beta^2 m^2 - A^2}{8(\beta+\Delta)^3\beta^2 m^2}$
$w_s{}^* = \frac{I_2^2+2I_8}{2(\beta+\Delta)} + \frac{2I_{11}\beta(\beta+\Delta)-A^2}{8\beta^2(\beta+\Delta)^3 m^2}$</td></tr>
<tr><td>$m \geq m_1$</td><td colspan="3">(1) $w_c{}^* = \frac{I_1^2}{2\beta}$, $w_s{}^* = \frac{I_2(2p+I_2)}{2\beta}$</td></tr>
<tr><td>Notes</td><td colspan="3">$I_7 = I_1\left[2(1-\eta)p + I_1\right]$ $I_8 = I_2 p - (\beta+\Delta)g$ $I_9 = (1-x_1)\left[4I_2^2\beta^2(\beta+\Delta)^2 m^2 - A^2(1-x_1)^2\right]$
$I_{10} = 4I_2^2 I_5^2\beta^2 - I_6^2 A^2$ $I_{11} = (1+\alpha)Amp$</td></tr>
</table>

We now proceed by defining social welfare so that we can investigate the effect of review manipulation on society as a whole. It is the sum of consumer welfare, the seller's profit, and the platform's profit, i.e., $w_s = w_c + \pi_p + \pi_s$. Substituting the results from Proposition 1 into (6), we obtain consumers' welfare w_c and social welfare w_s in Table 5. Table 5 presents an innovative analysis of the impact of platform regulatory penalty, audit cost, and manipulation cost on the equilibrium consumer welfare and social welfare by considering how it depends on the different thresholds.

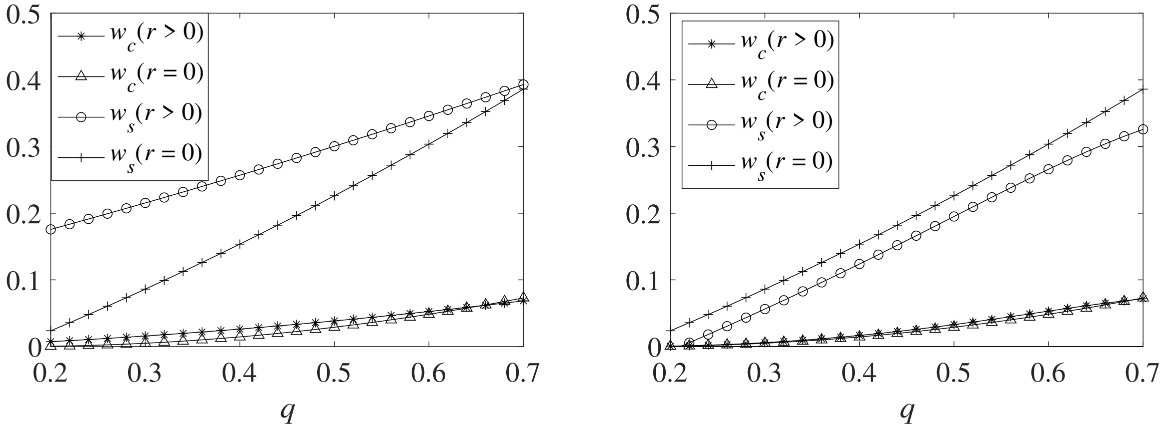
Next, we analyze the impact of online review manipulation on the consumer, the seller, the platform, and society. As the profit and welfare functions are too complex, it is impossible to derive analytical results. For this reason, we use numerical examples in our analysis. We have chosen different parameters to illustrate our analysis. Figure 8 and Figure 9 illustrate how D^*, π_p^*, π_s^*, w_c^* and w_s^* change as a function of q, when the seller manipulates reviews, and when he does not ($r > 0$ represents "manipulated" and $r = 0$ represents "unmanipulated").

Figure 8 (a) indicates that if the platform suffers a low regulatory penalty (e.g., $g = 0.005$), as q increases, then demand D and the profits increase as well. Additionally, Figure 8 (b) indicates that



(a) $g = 0.005$ (b) $g = 0.075$

Figure 8: Sensitivity analysis of D, $\pi_{\rm p}$ and $\pi_{\rm s}$ concerning q when $g = 0.005$ and $g = 0.075$



(a) $g = 0.005$ (b) $g = 0.075$

Figure 9: Sensitivity analysis of $w_{\rm c}$ and $w_{\rm s}$ concerning q when $g = 0.015$ and $g = 0.075$

the platform's profit under review manipulation $\pi_{\rm p}^*(r > 0)$ decreases if she suffers a high regulatory penalty (e.g., $g = 0.075$).

Moreover, in Figure 8 when the platform suffers a low regulatory penalty ($g = 0.005$), compared with the no-manipulation case, the demand, and both profits are all higher if the seller manipulates reviews (benefiting both players). However, when the platform suffers a high regulatory penalty ($g = 0.075$), compared with the no-manipulation case, the platform's profit decreases if the seller manipulates reviews. *Although manipulating online reviews brings more sales and benefits the seller, it hurts the platform's profit if the regulatory penalty is significant.* Figure 8 also indicates that the seller does not manipulate when the consumers' perceived quality revealed by unmanipulated reviews (q) is sufficiently large, further verifying the results in Figure 4.

From Figure 9, it follows that as q increases, both consumer welfare w_c and social welfare w_s increase. Moreover, when $g = 0.005$ (compared with the no manipulation case), consumer and social welfare are higher when the seller manipulates. *Surprisingly, seller's dishonest behavior creates social value, as consumers purchase a product they like but would not have tried otherwise.* When $g = 0.075$, consumer welfare is higher, but social welfare is lower if the seller manipulates online reviews. This is because *the increased consumer welfare and seller's profit due to the manipulation cannot balance out the platform's revenue loss when g is large.*

6. Considering an Endogenous Retail Price

This section considers the scenario in which the retail price is endogenous. The sequence of events is as follows: the platform sets the audit effort x; then, after observing x, the seller makes his manipulation and pricing decisions (r and p); finally, the consumers make their purchasing decisions. The platform's and seller's profits are represented by equations (7) and (8), respectively.

$$\pi_p(r,p,x) = p\alpha D(r,p,x) - (1-x)g\mathbb{I}_r - \mathbb{I}_r xp\alpha\eta\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - kx^2 \tag{7}$$

$$\pi_s(r,p,x) = p(1-\alpha)D(r,p,x) - \mathbb{I}_r xp(1-\alpha)\eta\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - mr^2 \tag{8}$$

We start with no audit case. Proposition 5 summarizes the equilibrium of the game (derived using backward induction) when the platform does not perform audits.

Proposition 5. *If the platform does not conduct audits (i.e., $x = 0$), only when $m \geq \hat{m}$, is $\pi_s(r,p)$ a joint concave function of (r,p), in which $\hat{m} = \frac{(1-\alpha)(1-\beta-\Delta)^2}{4(\beta+\Delta)}$. The equilibrium is represented by (9) to (12).*

$$p^* = \frac{2(\beta+\Delta)m\,(\beta+\Delta+(1-\beta-\Delta)\,q)}{4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^2} \tag{9}$$

$$r^* = \frac{(1-\alpha)\,(1-\beta-\Delta)\,(\beta+\Delta+(1-\beta-\Delta)\,q)}{4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^2} \tag{10}$$

$$\pi_s^* = \frac{(1-\alpha)m\,(\beta+\Delta+(1-\beta-\Delta)\,q)^2}{4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^2} \tag{11}$$

$$\pi_p^* = \frac{4\alpha(\beta+\Delta)m^2\,(\beta+\Delta+(1-\beta-\Delta)\,q)^2}{\left(4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^2\right)^2} - g \tag{12}$$

According to (9) to (10), it is evident that optimal price is a linear function of manipulation effort. *Corollary 4 proves that the higher the manipulation effort, the higher the retail price.* This is because a) the seller charges a premium for a product with high ratings, and b) the seller raises the retail price to cover the rising manipulation cost. As a result, the seller passes the manipulation cost to the consumer.

Corollary 4. *Let $x^* = 0$, then $p^* = \frac{2m(\beta+\Delta)r^*}{(1-\alpha)(1-\beta-\Delta)}$, the higher the r^* the higher the p^*.*

Figure 10 illustrates the findings of Corollary 4, plotting the relationship between p^* and r^* for different levels of consumers' intolerance to fake reviews. It indicates that the higher the consumers' intolerance to fake reviews, Δ, the steeper the p^* curve is. Consequently, the seller charges a higher premium due to manipulation. Thus, surprisingly, the more intolerant consumers are to fake reviews, the more the manipulation hurts them as they pay higher retail prices.

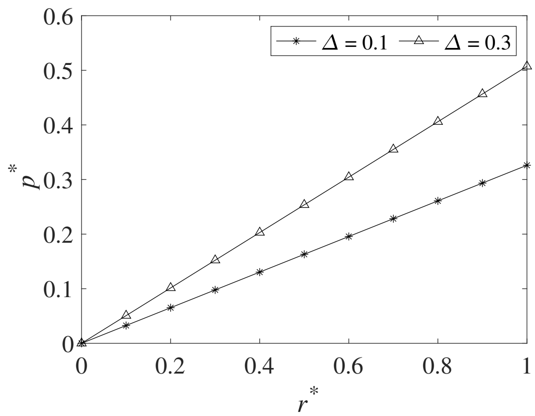


Figure 10: The relationship between p^* and r^*

Next, we derive Corollary 5 by evaluating the seller's profit with and without manipulation. It proves that for low-quality products, as revealed by unmanipulated reviews (i.e., q is small), the seller manipulates online reviews regardless of the manipulation cost. For the high-quality products shown by unmanipulated reviews (i.e., q is large), the seller manipulates reviews only if the manipulation cost is negligible. Moreover, for high-quality products ($q > q_2$) with high manipulation costs ($m > m_2$), the seller does not manipulate online reviews, even if the platform does not audit.

Corollary 5. *Let* $q_2 = \frac{\sqrt{\beta(\beta+\Delta)}}{1+\sqrt{\beta(\beta+\Delta)}}$ *and* $m_2 = \frac{(1-\alpha)(1-\beta-\Delta)^2[\beta+(1-\beta)q]^2}{4\Delta\left[q^2-\beta(\beta+\Delta)(1-q)^2\right]}$. *If the platform does not conduct audits, then when the pricing policy is endogenous: (1) if* $q \leq q_2$ *the seller manipulates reviews for any given* m*; (2) Otherwise, the seller manipulates reviews only if* $m \leq m_2$.

Having completed the analysis of the specific no-audit case, we proceed with the analysis of the general model of platform auditing. If the platform conducts an audit (i.e., $x > 0$), then the seller's best response is summarized in Proposition 6.

Proposition 6. *Let* $x > 0$. *Only when* $m \geq \dot{m}$, *is* $\pi_s(r, p)$ *a joint concave function of* (r, p), *in which* $\dot{m} = \frac{(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2}{4(\beta+\Delta)[(\beta+\Delta)(1-\eta)x+\beta(1-x)]}$. *For all* $0 < x < 1$, *the seller's best response is represented by (13) and (14).*

$$r(x) = \frac{(1-\alpha)(1-\beta-\Delta)(1-x)\{\beta[\beta+\Delta+(1-\beta-\Delta)q]-(\beta+\Delta)\eta x[\beta+(1-\beta)q]+\Delta qx\}}{4(\beta+\Delta)m[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2} \tag{13}$$

$$p(x) = \frac{2(\beta+\Delta)m\{\beta[\beta+\Delta+(1-\beta-\Delta)q]-(\beta+\Delta)\eta x[\beta+(1-\beta)q]+\Delta qx\}}{4(\beta+\Delta)m[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2} \tag{14}$$

Corollary 6. *Everything else is constant; the higher the* x, *the lower the* $p(x)$ *and* $r(x)$.

Corollary 6 proves that as in section 4 (when the price is exogenous), *the higher the platform's audit effort, the lower the seller's manipulation effort.* Counter-intuitively, Corollary 6 also indicates that the higher the platform's audit effort, the lower the retail price, as the seller cannot increase the perceived value of the product by manipulating reviews. For this reason, the demand shifts down, and the marginal revenue of the seller is at a lower price point. Substituting (13) and (14) into (7), we derive equation (15) for the platform's profit function.

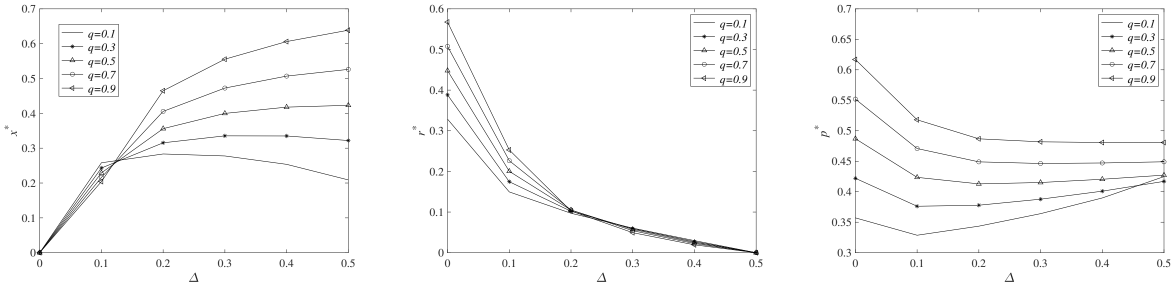
$$\begin{aligned}\pi_p(x) = & \frac{(\beta((\beta+\Delta)(1-q)+q)-(\beta+\Delta)\eta x(\beta+(1-\beta)q)+\Delta qx)^2}{\beta\left(4(\beta+\Delta)m((\beta+\Delta)(1-\eta)x+\beta(1-x))-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2\right)^2} \\ & \times 4\alpha m^2(\beta+\Delta)[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-g(1-x)-kx^2\end{aligned} \tag{15}$$

As (15) is a complex function of x, it is not possible to calculate the analytical solution of x^*, but we can analyze its local properties. Figure 11, obtained through numerical analysis, summarizes the effect of Δ on equilibrium with different q. Note that as $0 \leq \beta + \Delta \leq 1$, the x-axis is in the range [0, 0.5]. Figure 11 (a) shows that the impact of consumer intolerance for fake reviews (Δ) on the audit strategy (x^*) is concave and tends to increase with q, when $\Delta > 0.15$. For this reason, due to the combination of higher Δ and higher x^*, the seller reduces the manipulation of online reviews, as shown in Figure 11 (b).

Furthermore, in the context of endogenous pricing, as illustrated in Figure 11 (c), the impact of consumers' intolerance for fake reviews (Δ) on retail prices exhibits a convex shape. The underlying rationale is as follows. On one hand, sellers reduce prices due to heightened audit levels, as demonstrated in Corollary 6. On the other hand, sellers increase prices to maximize their profits. The former effect dominates when Δ is small, while the latter prevails when Δ is large. Additionally, on average, the higher the quality the higher the retail price.

Moreover, we have further done sensitivity analyses on the interaction between Δ and g, k, m, and η, as summarized in Figures C1 to C4 in Appendix C. These results corroborate the insights obtained from Figure 11. The impact of consumer intolerance for fake reviews (Δ) interacts with parameters (g, k, m, and η) in a consistent manner. The graphs exhibit similar patterns: the impact of Δ on the audit strategy (x^*) increases with Δ at decreasing rates; the impact of Δ on the manipulation effort (r^*) decreases with Δ at decreasing rates; the impact of Δ on the retail price (p^*) is convex. In general, prices are higher for lower and higher values of Δ, with a minimum price in the middle range of Δ.

Additionally, the results are also evidence that, for a given Δ, the impact of the parameter (g, k, m, and η) on the optimal policy differs across parameters. These findings align with the analytical and numerical results presented in the article.



(a) (b) (c)

Figure 11: The impact of Δ on optimal solutions with different q ($\eta = 0.05$, $k = 0.03$, $m = 0.5$, $g = 0.024$)

7. Summary and Discussion

This study focuses on the auditing and manipulation game between the platform and the seller. Figure 12 depicts the deductive process used to derive and analyze the equilibrium solution of the game.

As shown in Figure 12, we first solve the game by backward induction, step-by-step, from the consumer's decision to the seller's decision and finally to the platform's decision in Section 4. Then, in Section 5, we identify the major determinants of optimal policies derived in Section 4 by analyzing how the parameters impact equilibrium policies, profitability, and consumer and social welfare. Finally, in Section 6, we extend the main model from the exogenous retail price to the seller deciding the retail price endogenously by using backward induction and numerical analysis.

Next, Figure 13 represents how the lemmas, propositions, and corollaries in Figure 12 answer the research questions in Section 1.

The first question, "Under which conditions does the platform's self-regulation mechanism improve the quality of consumer information?" is answered by Lemma 2, Corollary 1, Figure 3, Proposition 2, Figure 6, and Corollary 6: rigorous auditing and expensive manipulation costs improve the quality of consumer information.

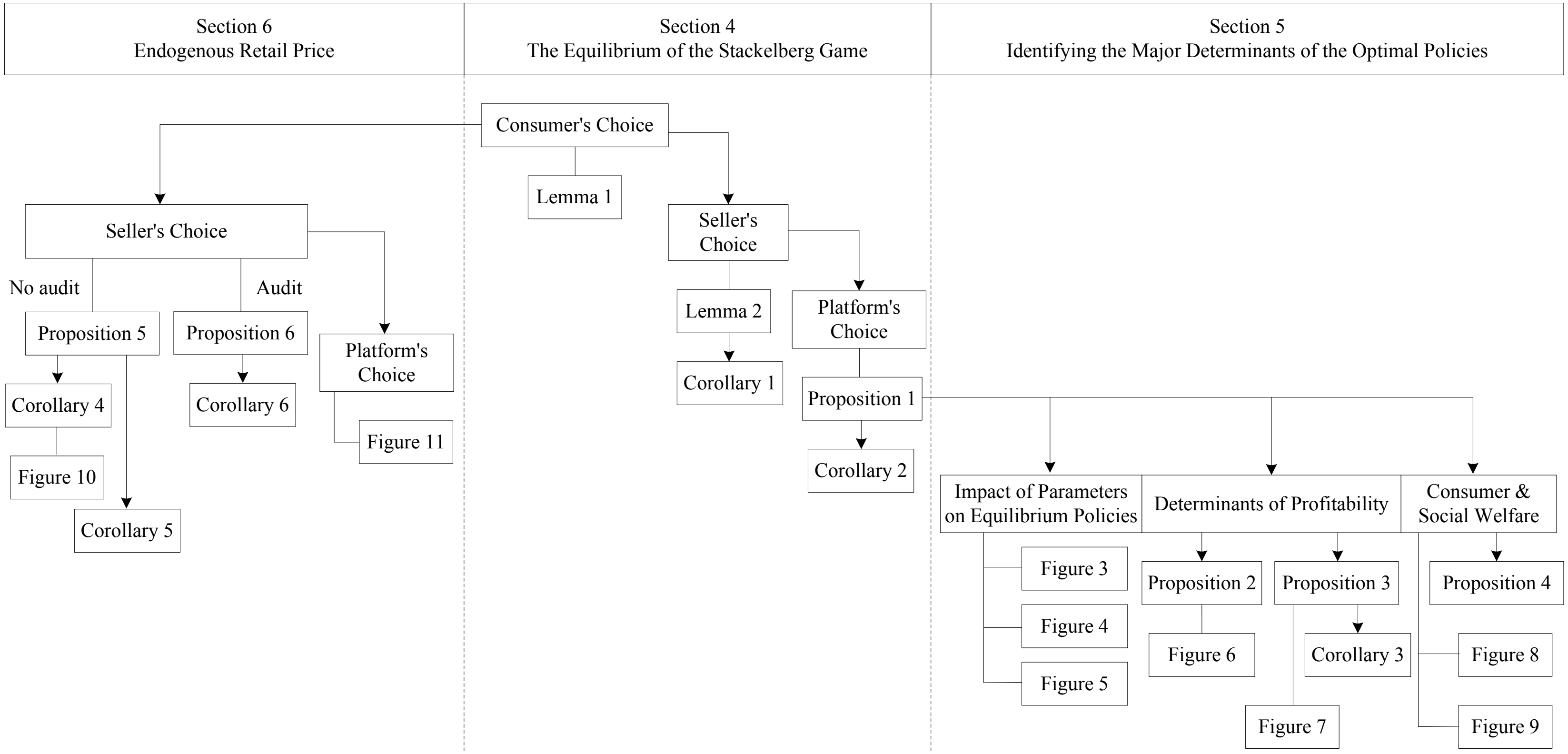


Figure 12: Flow chart depicting the structure of the deductive process.

The second question, “What are the incentives for the platform to audit the seller and for the seller to manipulate online reviews?” is answered by Lemma 2, Corollarys 1 to 3, Corollary 5, Proposition 2, Proposition 3, Figures 3 to 7, and Figure 11: the platform audits online reviews to prevent the high revenue loss caused by fake reviews, the seller manipulate online reviews due to low product quality, high retail price, and low costs of manipulation.

The third question, “What is the role of consumers in determining the quality of online reviews?” is answered by Lemma 1, Figure 3, Proposition 2, Figure 6, Proposition 3, Corollary 3, Figure 7, Figure 11, and Proposition 4: consumer activism and external supervision are the key drivers in determining the quality of online reviews.

The fourth question, “How does online review manipulation affect consumer and social welfare?” is answered by Proposition 4, Figure 8, and Figure 9: contrary to intuition, manipulating online reviews does not always reduce consumer and social welfare. In fact, it can increase consumer and social welfare when the platform suffers a negligibly low regulatory penalty due to fake reviews. This is because the manipulation can lead to more purchases.

Finally, the last question, “How do online review manipulation and platform audits affect the retail price?” is answered by Corollary 4, Corollary 6, Figure 10, and Figure 11: manipulation leads to higher retail prices, while audits lead to lower prices.

8. Conclusion

As sellers may manipulate online reviews to increase sales, platforms conduct audits and impose penalties on review manipulation to protect consumers from misleading information. In this article, using a Stackelberg game, we studied the auditing strategies of a platform (leader) and reviewed manipulation by a seller (follower). When the platform detects the manipulation, it removes the fake reviews. Consequently, the seller loses a fraction of future business, and the platform suffers an expected revenue loss due to the loss of the seller’s income. If the manipulation is undetected, the platform suffers regulatory penalty, and consumers reduce their trust in online reviews.

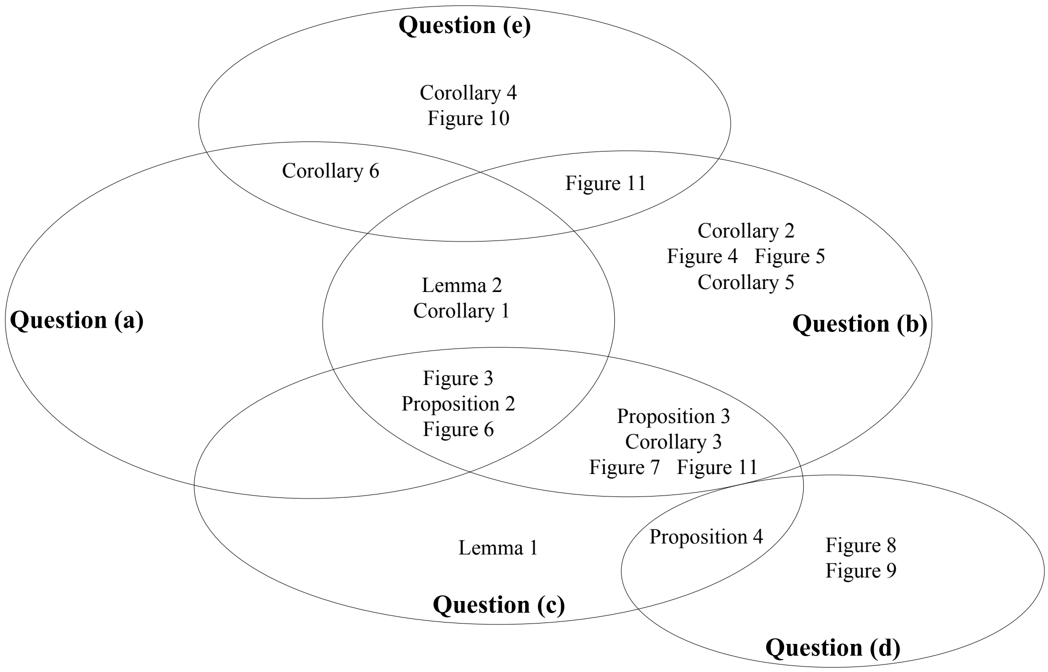


Figure 13: Venn diagram mapping the analytical and numerical results to the research questions.

Our central managerial insight is that the self-regulatory system has several perverse incentives and failures that lead to the maintenance and expansion of fake reviews. Moreover, consumer activism and intolerance of phony information are the key drivers, without which the platform fails to act, and the seller promotes fake reviews. Following is a summary of our major conclusions.

(1) It is clear that rigorous auditing and expensive costs effectively reduce online review manipulation by the seller and thus improve the quality of consumer information. Nonetheless, by default, the platform does not audit, even when the cost of so doing is very low, which leads to a high manipulation effort by the seller. The platform audits treviews only when the damage caused by fake reviews is sufficiently large.

(2) Self-regulation's Achilles' heel. Self-regulation fails because the platform may lose from auditing manipulation activities. This is because the higher the seller's penalty loss, the higher the platform's loss if the seller gets caught manipulating reviews, and the lower the audit effort. For this reason, in a scenario where manipulation is increasingly essential to the seller, the platform is

less likely to audit him. The platform is also exposed to more significant losses if she catches the seller manipulating online reviews.

(3) The audit policy is influenced by factors such as the audit cost, shared revenue loss from the punished seller, regulatory penalty, and anticipated manipulation effort by the seller. For instance, a higher audit cost and shared revenue loss from the punished seller lead to a looser audit, while a higher regulatory penalty and anticipated manipulation effort by the seller lead to a stricter audit. Additionally, sellers manipulate more for low-quality or high-price products when the platform has a lax audit policy, the manipulation is cheap, or consumers have a high tolerance for fake reviews.

(4) Consumer activism and external supervision serve as the primary factors influencing the quality of the review system. When the platform is subject to stringent external supervision, it incurs a substantial regulatory penalty in the presence of fake reviews, leading to an escalation in the auditing effort. Furthermore, when consumers significantly diminish their dependence on the seller's information due to the presence of fake reviews, the seller loses the incentive to manipulate reviews. The cumulative effect of an increase in the regulatory penalty and the diminished effectiveness of online reviews results in an enhancement in the quality of the information provided to consumers.

(5) The paradox of fake reviews. We discuss how manipulating online reviews under some conditions increases consumers' expected product quality and, under certain conditions, social welfare increases as well. This result is interesting: review manipulation leads consumers to try a product they like, which is usually perceived as dishonest behavior and increases social welfare. Nonetheless, manipulating online reviews decreases social welfare if the regulatory penalty is significant. Hence, in this case, the platform must have an anti-manipulation policy.

(6) When the retail price is endogenous, i.e., the seller is not a price taker, we cannot derive a closed-form solution. When the platform does not audit, an analytical solution exists: the higher the manipulation effort, the higher the price (and vice-versa). The numerical results prove that the equilibrium relationship between pricing, auditing, and manipulation is highly complex. We found

evidence supporting that endogenous retail prices lead to a higher (lower) audit and lower (higher) manipulation, depending on product quality.

There are many other exciting extensions for future research. First, we have considered one seller selling products through one platform. In reality, many sellers sell homogeneous products on a platform. Hence, it would be interesting to study the impact of a platform's audits and penalties on sellers in a competitive market. Second, we consider that the seller loses part of future business if the manipulation is detected. However, platforms' penalty strategies are complex. Thus, it would be interesting to consider different penalties. Third, studying the dynamic interaction between a platform's audit and a seller's manipulation in a multi-period setting would be another exciting extension of our work.

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Appendices

Appendix A. Notation & Thresholds

This section summarizes the notation in Table A1 and the calculation of the different thresholds used in the article, in Table A2.

Table A1
Notation

Decision variables:	
x	Audit effort
r	Manipulation effort
Auxiliary variables:	
D	Expected demand
u	Consumer expected net utility
w_c	Consumer welfare
w_s	Social welfare
π_p	The platform's profit
π_s	The seller's profit
Parameters:	
p	Selling price
α	Agency fee
q	Consumer's perceived quality revealed by unmanipulated online review information
$\tilde{q}$	Consumers perceived quality revealed by idiosyncratic information
β	The weight of $\tilde{q}$ in consumers' expected quality of the product without fake reviews
Δ	Consumers' intolerance for fake reviews
k	Audit cost coefficient
m	Manipulation cost coefficient
g	The regulatory penalty
η	The seller's penalty loss due to failed audits

Table A2
Thresholds

Notation	Definition
g_1	$\frac{\alpha pA-mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$
m_0	$\frac{\alpha(1-\alpha)(1-\beta-\Delta)^2 p^2}{2(\beta+\Delta)^2 k}$
g_2	$\frac{2x_1 km\beta(\beta+\Delta)^2+(1-x_1)\alpha pA-mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$
m_1	$\frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4(\beta+\Delta)(q-p)\Delta}$
g_3	$\frac{\alpha p\big(Ax_2+2m(\beta+\Delta)B\big)+2\beta(\beta+\Delta)^2 mx_1 k}{2\beta(\beta+\Delta)^2 m}$
k_1	$\frac{2\alpha\Delta p(q-p)}{\beta(\beta+\Delta)}$
x_1	$\frac{(A-2mB(\beta+\Delta)-2(\beta+\Delta)\sqrt{m\eta A[q-p+\beta(1-q)]+m^2B^2})}{A}$

Appendix B. Mathematical Proofs

Proof of Lemma 1: By comparing the demand with and without manipulation, we find that $D(r,x\,|\,\mathbb{1}_r=1)-D(r,x\,|\,\mathbb{1}_r=0)=\frac{(1-x)[\beta r(1-\beta-\Delta)-\Delta(q-p)]}{\beta(\beta+\Delta)}$. As $0<\beta<\beta+\Delta<1$, thus: (1) if $q<p$ then $D(r,x\,|\,\mathbb{1}_r=1)>D(r,x\,|\,\mathbb{1}_r=0)$; (2) if $q\geq p$ then $D(r,x\,|\,\mathbb{1}_r=1)>D(r,x\,|\,\mathbb{1}_r=0)$ when $r>\frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$, and $D(r,x\,|\,\mathbb{1}_r=1)\leq D(r,x\,|\,\mathbb{1}_r=0)$ when $r\leq\frac{\Delta(q-p)}{\beta(1-\beta-\Delta)}$. □

Proof of Lemma 2: If we assume an interior point solution such that $\mathbb{I}_r = 1$, it is easily verifiable that $\pi_s(r, x \,|\, \mathbb{I}_r = 1)$ is concave in r. From the first-order condition $\partial\pi_s(r, x|\mathbb{I}_r = 1)/\partial r = 0$, we obtain (A1).

$$\hat{r} = \frac{(1-\alpha)(1-x)(1-\beta-\Delta)\,p}{2m(\beta+\Delta)} \tag{A1}$$

Next, substituting (A1) into equation (3), then the difference between the seller's profit from manipulating online reviews and not manipulating online reviews is

$$\pi_s(r=\hat{r}, x) - \pi_s(r=0, x) = \frac{(1-\alpha)p}{4\beta(\beta+\Delta)^2 m}\left[Ax^2 - 2x\left(A - 2m(\beta+\Delta)B\right) + A + 4m(\beta+\Delta)\Delta(p-q)\right]$$

where, $A = p\beta(1-\alpha)(1-\beta-\Delta)^2 > 0$, $B = (p-q)((\beta+\Delta)\eta - \Delta) - (1-q)\beta(\beta+\Delta)\eta$.

As we assume that $q - p + \beta(1-q) > 0$, let $\pi_s(r=\hat{r}, x) - \pi_s(r=0, x) = 0$, we get two possible solutions, equations (A2) and (A3).

$$x_1 = \frac{A - 2m(\beta+\Delta)B - 2(\beta+\Delta)\sqrt{m\eta A\left[q-p+\beta(1-q)\right] + m^2B^2}}{A} \tag{A2}$$

$$x_2 = \frac{A - 2m(\beta+\Delta)B + 2(\beta+\Delta)\sqrt{m\eta A\left[q-p+\beta(1-q)\right] + m^2B^2}}{A} \tag{A3}$$

As $A > 0$, it follows that: a) if $x \le x_1$ or $x \ge x_2$ then $\pi_s(r=\hat{r}, x) \ge \pi_s(r=0, x)$ and, consequently, $r(x) = \hat{r}$; b) if $x_1 < x < x_2$ then $\pi_s(r=\hat{r}, x) < \pi_s(r=0, x)$ from which we obtain $r(x) = 0$. As $0 \le x \le 1$ and $x_2 > 1 > x_1$, we then conclude that the seller's best response is represented by (A4).

$$r(x) = \begin{cases} \frac{(1-\alpha)(1-x)(1-\beta-\Delta)p}{2m(\beta+\Delta)} & if\ x_1 > 0\ and\ 0 \le x \le x_1 \\ 0 & if\ x_1 > 0\ and\ x > x_1 \\ 0 & if\ x_1 \le 0 \end{cases} \quad \square \tag{A4}$$

Proof of Corollary 1: when $p > q$ we have $B < 0$, thus $A - 2m(\beta + \Delta)B > 0$. Then, $x_1 > 0 \Leftrightarrow (A-2m(\beta+\Delta)B)^2 > 4(\beta+\Delta)^2(m\eta A[q-p+\beta(1-q)]+m^2B^2) \Leftrightarrow p\beta(1-\alpha)(1-\beta-\Delta)^2 > -4m(\beta + \Delta)\Delta(p - q)$. As $1 - \alpha > 0$, $\Delta > 0$, thus when $q < p$ we have $x_1 > 0$ for any given m. When $q > p$ we have $\frac{\partial x_1}{\partial m} = -\frac{(\beta+\Delta)(\sqrt{A\eta m(q-p+\beta(1-q))+B^2m^2}+Bm)^2}{Am\sqrt{A\eta m(q-p+\beta(1-q))+B^2m^2}} < 0$. Let $x_1 = 0$, we have $m = m_1 = \frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4(\beta+\Delta)(q-p)\Delta} > 0$. Thus, when $q > p$ and $m < m_1$, we have $x_1 > 0$; when $q > p$ and $m > m_1$, we have $x_1 < 0$. Thus, when $q > p$ and $m > m_1$, for any given audit effort x the seller's best response is $r(x) = 0$. □

Proof of Proposition 1: From (A4), the seller's best response could be rewritten by (A5).

$$r(x) = \begin{cases} \frac{(1-\alpha)(1-x)(1-\beta-\Delta)p}{2m(\beta+\Delta)} & if\ q \le p\ (or\ q > p,\ m < m_1)\ and\ 0 \le x \le x_1 \\ 0 & if\ q \le p\ (or\ q > p,\ m < m_1)\ and\ x > x_1 \\ 0 & if\ q > p,\ m \ge m_1 \end{cases} \tag{A5}$$

From (A5), we know that:

(1) When $x_1 \le 0$, i.e., $q > p$ and $m \ge m_1$, for any $x \in [0, 1]$, we have $\pi_s(r = \hat{r}, x) \le \pi_s(r = 0, x)$, thus the seller's best response is $r = 0$ and, for this reason, $\mathbb{I}_r = 0$, and $D(r = 0) = 1 - \frac{p-(1-\beta)q}{\beta}$. Therefore, the optimization problem of the platform is

$$\max_x \pi_p(x) = \max_x \left(p\alpha\left[1 - \frac{p-(1-\beta)q}{\beta}\right] - kx^2\right)$$

thus $x^* = 0$, $r^* = 0$.

(2) When $x_1 > 0$, i.e., $q < p$ or ($q > p$ and $m < m_1$), there are two kinds of situations:

a. If $x_1 < x \le 1$, we have $\pi_s(r = \hat{r}, x) < \pi_s(r = 0, x)$, thus $r^* = 0$. Therefore, the optimization problem of the platform is

$$\max_x \pi_p(x) = \max_x \left(p\alpha D(r = 0) - kx^2\right)$$
$$s.t.\ \ x_1 < x \le 1$$

thus x^* converges to x_1.

b. If $0 \leq x \leq x_1$, we have $\pi_s(r = \hat{r}, x) \geq \pi_s(r = 0, x)$, thus the seller's best response is $r(x) = \hat{r} = \frac{(1-\alpha)(1-x)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$. Substituting $r(x)$ into equation (5), the optimization problem of the platform is

$$\max_x \pi_p(x) = \max_x \left\{ p\alpha D(x, r(x)) - (1-x)g - xp\alpha\eta \left[1 - \frac{p-(1-\beta)q}{\beta}\right] - kx^2 \right\} \quad \text{(A6)}$$
$$s.t.\ \ 0 \leq x \leq x_1$$

from which we derive $\frac{\partial^2 \pi_p(x)}{\partial x^2} = \frac{\alpha(1-\alpha)(1-\beta-\Delta)^2p^2 - 2(\beta+\Delta)^2 km}{(\beta+\Delta)^2 m} = \frac{\alpha Ap - 2km\beta(\beta+\Delta)^2}{\beta(\beta+\Delta)^2 m}$. Let $m_0 = \frac{\alpha(1-\alpha)(1-\beta-\Delta)^2p^2}{2(\beta+\Delta)^2 k}$, we find that when $m > m_0$ we have $\frac{\partial^2 \pi_p(x)}{\partial x^2} < 0$ and when $m \leq m_0$ we have $\frac{\partial^2 \pi_p(x)}{\partial x^2} \geq 0$.

(b1) when $m_0 < m < m_1$, we have $\frac{\partial^2 \pi_p(x)}{\partial x^2} < 0$, which means $\pi_p(x)$ is concave in x. Let $\frac{\partial \pi_p(x)}{\partial x} = 0$, we can get $\hat{x} = \frac{\beta(\beta+\Delta)^2 gm - \alpha pA + mp\alpha(\beta+\Delta)B}{2km\beta(\beta+\Delta)^2 - \alpha Ap}$.

Since $0 \leq x \leq x_1$, let $g_1 = \frac{\alpha pA - mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$, $g_2 = \frac{2x_1 km\beta(\beta+\Delta)^2 + (1-x_1)\alpha pA - mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$, thus when $g_1 < g < g_2$, i.e., $0 < \hat{x} < x_1$, we have $x^* = \hat{x} = \frac{\beta(\beta+\Delta)^2 gm - \alpha pA + mp\alpha(\beta+\Delta)B}{2km\beta(\beta+\Delta)^2 - \alpha Ap}$, $r^* = \frac{[(2k-g)\beta(\beta+\Delta) - p\alpha B]A}{2\beta(1-\beta-\Delta)(2km\beta(\beta+\Delta)^2 - \alpha Ap)}$; when $g \geq g_2$, i.e., $\hat{x} \geq x_1$, we have $x^* = x_1$, $r^* = 0$; when $0 \leq g \leq g_1$, i.e., $\hat{x} < 0$, we have $x^* = 0$, $r^* = \frac{(1-\alpha)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$.

(b2) when $0 \leq m \leq min\{m_0, m_1\}$, we have $\frac{\partial^2 \pi_p(x)}{\partial x^2} \geq 0$, which means $\pi_p(x)$ is convex in x. Thus, $x^* = 0$ *or* x_1. From equation (A2), we have $\pi_p(x = 0) - \pi_p(x = x_1) = \frac{x_1[\alpha p(Ax_2 + 2m(\beta+\Delta)B) + 2\beta(\beta+\Delta)^2 m(x_1 k - g)]}{2(\beta+\Delta)^2 \beta m}$. Moreover, let $g_3 = \frac{\alpha p(Ax_2 + 2m(\beta+\Delta)B) + 2\beta(\beta+\Delta)^2 mx_1 k}{2\beta(\beta+\Delta)^2 m}$, we can verify that $\pi_p(x = 0) \leq \pi_p(x = x_1)$ when $g \geq g_3$, $\pi_p(x = 0) > \pi_p(x = x_1)$ when $0 \leq g < g_3$. Thus, when $g \geq g_3$ we have $x^* = x_1$ and $r^* = \frac{(1-\alpha)(1-x_1)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$, when $0 \leq g < g_3$ we have $x^* = 0$ and $r^* = \frac{(1-\alpha)(1-\beta-\Delta)p}{2m(\beta+\Delta)}$. □

Proof of Corollary 2: It is easily verifiable that $\pi_s(\mathbb{I}_r = 1, x = 0) = p(1-\alpha)\left[1 - \frac{p-(1-\beta-\Delta)(q+r)}{\beta+\Delta}\right] - mr^2$ is concave in r. From the first-order condition $d\pi_s(\mathbb{I}_r = 1, x = 0)/dr = 0$, we can obtain $\hat{r} = \frac{(1-\alpha)(1-\beta-\Delta)p}{2(\beta+\Delta)m}$. Substituting $\hat{r}$ into $\pi_s(\mathbb{I}_r = 1, x = 0)$, by evaluating the difference in $\pi_s(\mathbb{I}_r = 1, x = 0)$ and $\pi_s(\mathbb{I}_r = 0, x = 0)$, we find that only if $p(1-\alpha)\left[4(\beta+\Delta)m\Delta(p-q) + p\beta(1-\alpha)(1-\beta-\Delta)^2\right] >$

0, i.e., $q < p + \frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4m(\beta+\Delta)\Delta}$ then $\pi_s(\mathbb{I}_r = 1, x = 0) > \pi_s(\mathbb{I}_r = 0, x = 0)$. Otherwise, if $q \geq p + \frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4m(\beta+\Delta)\Delta}$ then $\pi_s(\mathbb{I}_r = 1, x = 0) \leq \pi_s(\mathbb{I}_r = 0, x = 0)$. □

Proof of the characteristics of threshold lines g_1, g_2, g_3, m_0, and m_1 in Figure 3: From Tables 2 and 3 we get the following threshold lines: $g_1 = \frac{\alpha pA - mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$, $g_2 = \frac{2x_1 km\beta(\beta+\Delta)^2 + (1-x_1)\alpha pA - mp\alpha(\beta+\Delta)B}{\beta(\beta+\Delta)^2 m}$, $g_3 = \frac{\alpha p(Ax_2 + 2m(\beta+\Delta)B) + 2\beta(\beta+\Delta)^2 mx_1 k}{2\beta(\beta+\Delta)^2 m}$, $m_0 = \frac{\alpha(1-\alpha)(1-\beta-\Delta)^2 p^2}{2(\beta+\Delta)^2 k}$, $m_1 = \frac{p\beta(1-\alpha)(1-\beta-\Delta)^2}{4(\beta+\Delta)(q-p)\Delta}$.

(1) Taking the first-order derivative of g_1, g_2 and g_3 with respect to m, we have: $\frac{\partial g_1}{\partial m} = -\frac{\alpha Ap}{\beta(\beta+\Delta)^2 m^2} < 0$, $\frac{\partial g_2}{\partial m} = -\frac{\alpha A^2\eta mp[q-p+\beta(1-q)] + 2\beta(\beta+\Delta)^2 km(\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}+Bm)^2}{A\beta(\beta+\Delta)m^2\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}} < 0$, $\frac{\partial g_3}{\partial m} = -\frac{\alpha A^2\eta mp[q-p+\beta(1-q)] + 2\beta(\beta+\Delta)^2 km(\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}+Bm)^2}{2A\beta(\beta+\Delta)m^2\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}} - \frac{\alpha Ap}{2\beta(\beta+\Delta)^2 m^2} < 0$.

(2) Substituting $m = m_0$ into g_1, g_2 and g_3, then we can verify that $g_1 = g_2 = g_3 = \frac{2\beta(\beta+\Delta)k - \alpha pB}{\beta(\beta+\Delta)}$ if $m = m_0$.

(3) $g_2 - g_1 = \frac{x_1\left[2\beta km(\beta+\Delta)^2 - \alpha Ap\right]}{\beta m(\beta+\Delta)^2}$, thus $g_2 > g_1$ if $m > m_0$.

(4) Comparing the difference between m_1 and m_0, we can verify that

$m_1 - m_0 = \frac{p(1-\alpha)(1-\beta-\Delta)^2[\beta(\beta+\Delta)k - 2\alpha\Delta p(q-p)]}{4(\beta+\Delta)^2\Delta k(q-p)}$. Thus, there is a threshold value $k_1 = \frac{2p\alpha\Delta(q-p)}{(\beta+\Delta)\beta}$ such that $m_1 > m_0$ when $k > k_1$, and $m_1 < m_0$ when $k < k_1$.

Thus, the threshold lines g_1, g_2, g_3, m_0, and m_1 have the following characteristics: (1) $\partial g_1/\partial m < 0$, $\partial g_2/\partial m < 0$, $\partial g_3/\partial m < 0$; (2) $g_1 = g_2 = g_3$ if $m = m_0$; (3) $g_2 > g_1$ if $m > m_0$; (4) $m_1 > m_0$ if and only if $k > k_1$, and $m_1 < m_0$ if and only if $k < k_1$. □

Proof of the influence of parameters k and η on the threshold lines g_1, g_2, g_3, m_0, and m_1 in Figure 3: Taking the first-order derivatives of m_0, m_1, g_1, g_2 and g_3 with respect to k, we have: $\frac{\partial m_0}{\partial k} = -\frac{\alpha Ap}{2\beta(\beta+\Delta)^2 k^2} < 0$, $\frac{\partial m_1}{\partial k} = 0$, $\frac{\partial g_1}{\partial k} = 0$, $\frac{\partial g_2}{\partial k} = 2x_1 > 0$, $\frac{\partial g_3}{\partial k} = x_1 > 0$.

Taking the first-order derivatives of m_0, m_1, g_1, g_2 and g_3 with respect to η, we have: $\frac{\partial m_0}{\partial \eta} = 0$, $\frac{\partial m_1}{\partial \eta} = 0$, $\frac{\partial g_1}{\partial \eta} = \frac{\alpha p[q-p+\beta(1-q)]}{\beta} > 0$,
$\frac{\partial g_2}{\partial \eta} = \frac{[q-p+\beta(1-q)]\{(\beta+\Delta)(4\beta(\beta+\Delta)^2 km - \alpha Ap)\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2} - (A - 2(\beta+\Delta)Bm)(2\beta(\beta+\Delta)^2 km - \alpha Ap)\}}{A\beta(\beta+\Delta)\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}$,
$\frac{\partial g_3}{\partial \eta} = \frac{[q-p+\beta(1-q)]\{4\beta(\beta+\Delta)^3 km\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2} + (A - 2(\beta+\Delta)Bm)(\alpha Ap - 2\beta(\beta+\Delta)^2 km)\}}{2A\beta(\beta+\Delta)\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}$. When $m < m_0$, we have $\alpha Ap - 2\beta(\beta+\Delta)^2 km > 0$. Besides $x_1 > 0$ when $m < m_0$, then we have $A - 2(\beta+\Delta)Bm > 0$. Thus $\frac{\partial g_3}{\partial \eta} > 0$, but $\frac{\partial g_2}{\partial \eta}$ may be positive or negative.

Taking the second-order derivative of g_2 with respect to η, we have $\frac{\partial^2 g_2}{\partial \eta^2} = \frac{Ax_1x_2m[q-p+\beta(1-q)]^2(2\beta(\beta+\Delta)^2km-\alpha Ap)}{2\beta(\beta+\Delta)\{A\eta m[q-p+\beta(1-q)]+B^2m^2\}^{3/2}}$. As when $m > m_0$ we have $2\beta(\beta+\Delta)^2km - \alpha Ap > 0$, thus $\frac{\partial^2 g_2}{\partial \eta^2} > 0$, i.e., g_2 is convex function of η.

Thus, the influence of parameters k and η on the threshold lines g_1, g_2, g_3, m_0, and m_1 is as follows: (1) $\partial m_0/\partial k < 0$, $\partial m_1/\partial k = 0$, $\partial g_1/\partial k = 0$, $\partial g_2/\partial k > 0$, $\partial g_3/\partial k > 0$; (2) $\partial m_0/\partial \eta = 0$, $\partial m_1/\partial \eta = 0$, $\partial g_1/\partial \eta > 0$, $\partial g_3/\partial \eta > 0$, $\partial^2 g_2/\partial \eta^2 > 0$. □

Proof of Proposition 2: By taking the first-order derivatives of x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$ with respect to g: A) From the optimal solution for high x^* and low r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial g} = 0$, $\frac{\partial r^*}{\partial g} = 0$, $\frac{\partial {\pi_s}^*}{\partial g} = 0$, $\frac{\partial {\pi_p}^*}{\partial g} = -\frac{2(\beta+\Delta)(\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}+Bm)}{A} < 0$. B) From the optimal solution for medium x^* and r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial g} = \frac{\beta(\beta+\Delta)^2 m}{2\beta(\beta+\Delta)^2km-\alpha Ap}$, $\frac{\partial r^*}{\partial g} = -\frac{(1-\alpha)\beta(1-\beta-\Delta)(\beta+\Delta)p}{2(2\beta(\beta+\Delta)^2km-\alpha Ap)}$, $\frac{\partial {\pi_p}^*}{\partial g} = \frac{(\beta+\Delta)m[\alpha Bp+\beta(\beta+\Delta)(g-2k)]}{2\beta(\beta+\Delta)^2km-\alpha Ap}$, $\frac{\partial {\pi_s}^*}{\partial g} = \frac{(1-\alpha)\beta(\beta+\Delta)^2mp[A(g-2k)+2(\beta+\Delta)Bkm]}{2(2\beta(\beta+\Delta)^2km-\alpha Ap)^2} + \frac{(1-\alpha)(\beta+\Delta)Bmp}{2(2\beta(\beta+\Delta)^2km-\alpha Ap)}$. As $2(2\beta(\beta+\Delta)^2km - \alpha Ap)$ when $m > m_0$, thus, $\frac{\partial x^*}{\partial g} > 0$, $\frac{\partial r^*}{\partial g} < 0$. As $g_1 < g < g_2$, we have $\frac{\partial {\pi_p}^*}{\partial g} < 0$, $\frac{\partial {\pi_s}^*}{\partial g} < -\frac{2(1-\alpha)(\beta+\Delta)p\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}{2(2\beta(\beta+\Delta)^2km-\alpha Ap)} < 0$. C) From the optimal solution for $x^* = 0$ and high r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial g} = 0$, $\frac{\partial r^*}{\partial g} = 0$, $\frac{\partial {\pi_s}^*}{\partial g} = 0$, $\frac{\partial {\pi_p}^*}{\partial g} = -1 < 0$.

Taking the first-order derivatives of x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$ with respect to η: A) From the optimal solution for high x^* and low r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial \eta} = -\frac{(\beta+\Delta)mx_1[q-p+\beta(1-q)]}{\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}} < 0$, $\frac{\partial r^*}{\partial \eta} = \frac{(1-\alpha)(1-\beta-\Delta)px_1[q-p+\beta(1-q)]}{2\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}} > 0$, $\frac{\partial \pi_s^*}{\partial \eta} = 0$, $\frac{\partial \pi_p^*}{\partial \eta} = -\frac{(\beta+\Delta)mx_1[q-p+\beta(1-q)][4\beta(\beta+\Delta)^2m(g-2kx_1)-2\alpha Ap(1-x_1)]}{4\beta(\beta+\Delta)^2m\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}$. As $g > g_2$, we have $4\beta(\beta+\Delta)^2m(g-2kx_1)-2\alpha Ap(1-x_1) > 4\alpha(\beta+\Delta)p\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2} > 0$, thus $\frac{\partial \pi_p^*}{\partial \eta} < 0$. B) From the optimal solution for medium x^* and r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial \eta} = -\frac{\alpha(\beta+\Delta)^2mp[q-p+\beta(1-q)]}{2\beta(\beta+\Delta)^2km-\alpha Ap} < 0$, $\frac{\partial r^*}{\partial \eta} = \frac{\alpha A(\beta+\Delta)p[q-p+\beta(1-q)]}{2\beta(1-\beta-\Delta)(2\beta(\beta+\Delta)^2km-\alpha Ap)} > 0$, $\frac{\partial \pi_p^*}{\partial \eta} = -\frac{\alpha p[q-p+\beta(1-q)](\alpha(\beta+\Delta)Bmp+\beta(\beta+\Delta)^2gm-\alpha Ap)}{\beta(2\beta(\beta+\Delta)^2km-\alpha Ap)} < 0$, $\frac{\partial {\pi_s}^*}{\partial \eta} = -p(1-\alpha)[q-p+\beta(1-q)]$ $\times\frac{2\alpha^2A^2p^2-3\alpha^2A(\beta+\Delta)Bmp^2-\alpha A\beta(\beta+\Delta)^2gmp-6\alpha A\beta(\beta+\Delta)^2kmp+8\alpha\beta(\beta+\Delta)^3Bkm^2p+4\beta^2(\beta+\Delta)^4gkm^2}{2\beta(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}$,
$\frac{\partial^2 {\pi_s}^*}{\partial \eta^2} = \frac{\alpha(1-\alpha)(\beta+\Delta)^2mp^2[\beta(1-q)+q-p]^2(8\beta(\beta+\Delta)^2km-3\alpha Ap)}{2\beta(2\beta(\beta+\Delta)^2km-\alpha Ap)^2} > 0$. Let $\frac{\partial {\pi_s}^*}{\partial \eta} = 0$, we have
$\bar{\eta} = \frac{(3\beta(\beta+\Delta)^2km-\alpha Ap)(\beta(\beta+\Delta)^2gm-2\alpha Ap)+\beta^2(\beta+\Delta)^4gkm^2}{\alpha(\beta+\Delta)^2mp[\beta(1-q)+q-p](8\beta(\beta+\Delta)^2km-3\alpha Ap)} + \frac{\Delta(q-p)}{(\beta+\Delta)[\beta(1-q)+q-p]}$. As $\frac{\partial^2 {\pi_s}^*}{\partial \eta^2} > 0$, we can verify that $\frac{\partial \pi_s^*}{\partial \eta} < 0$ if $\eta < \bar{\eta}$, and $\frac{\partial \pi_s^*}{\partial \eta} > 0$ if $\eta > \bar{\eta}$. C) From the optimal solution for $x^* = 0$ and high r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial \eta} = 0$, $\frac{\partial r^*}{\partial \eta} = 0$, $\frac{\partial \pi_s^*}{\partial \eta} = 0$, $\frac{\partial \pi_p^*}{\partial \eta} = 0$.

Taking the first-order derivatives of x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$ with respect to k: A) From the optimal solution for high x^* and low r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial k}=0$, $\frac{\partial r^*}{\partial k}=0$, $\frac{\partial {\pi_p}^*}{\partial k}=-{x_1}^2<0$, $\frac{\partial {\pi_s}^*}{\partial k}=0$. B) From the optimal solution for medium x^* and r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial k}=-\frac{2\beta(\beta+\Delta)^2m(\alpha(\beta+\Delta)Bmp+\beta(\beta+\Delta)^2gm-\alpha Ap)}{(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}<0$, $\frac{\partial r^*}{\partial k}=\frac{A(\beta+\Delta)(\alpha(\beta+\Delta)Bmp+\beta(\beta+\Delta)^2gm-\alpha Ap)}{(1-\beta-\Delta)(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}>0$, $\frac{\partial {\pi_p}^*}{\partial k}=-\frac{(\alpha(\beta+\Delta)Bmp+\beta(\beta+\Delta)^2gm-\alpha Ap)^2}{(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}<0$, $\frac{\partial {\pi_s}^*}{\partial k}=-(1-\alpha)(\beta+\Delta)mp(\alpha(\beta+\Delta)Bmp+\beta(\beta+\Delta)^2gm-\alpha Ap)\times\frac{4\beta(\beta+\Delta)^2Bkm-A[\alpha Bp+\beta(\beta+\Delta)(2k-g)]}{(2\beta(\beta+\Delta)^2km-\alpha Ap)^3}$. As $g_1<g<g_2$, we have $\frac{4\beta(\beta+\Delta)^2Bkm-A[\alpha Bp+\beta(\beta+\Delta)(2k-g)]}{(2\beta(\beta+\Delta)^2km-\alpha Ap)^3}<\frac{Ax_1-A+2(\beta+\Delta)Bm}{(\beta+\Delta)m(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}=-\frac{2(\beta+\Delta)\sqrt{A\eta m[\beta(1-q)+q-p]+B^2m^2}}{(\beta+\Delta)m(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}<0$, thus $\frac{\partial {\pi_s}^*}{\partial k}>0$. C) From the optimal solution for $x^*=0$ and high r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial k}=0$, $\frac{\partial r^*}{\partial k}=0$, $\frac{\partial {\pi_p}^*}{\partial k}=0$, $\frac{\partial {\pi_s}^*}{\partial k}=0$.

Taking the first-order derivatives of x^*, r^*, ${\pi_p}^*$ and ${\pi_s}^*$ with respect to m: A) From the optimal solution for high x^* and low r^* in Tables 2 and 3, we have: $\frac{\partial x^*}{\partial m}=-\frac{(\beta+\Delta)(\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}+Bm)^2}{Am\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}<0$, $\frac{\partial x^*}{\partial m}=-\frac{(1-\alpha)(1-\beta-\Delta)\eta p[q-p+\beta(1-q)]}{2m\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}<0$, $\frac{\partial {\pi_s}^*}{\partial m}=0$, $\frac{\partial {\pi_p}^*}{\partial m}=-\frac{\alpha Ap(1-x_1)^2}{4\beta(\beta+\Delta)^2m^2}$
$-\frac{(\beta+\Delta)(\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}+Bm)^2[4\beta(\beta+\Delta)^2m(g-2kx_1)-2\alpha Ap(1-x_1)]}{4A\beta(\beta+\Delta)^2m^2\sqrt{A\eta m[q-p+\beta(1-q)]+B^2m^2}}<0$.

B) From the optimal solution for medium x^* and r^* in Tables 2 and 3, we have:

$\frac{\partial x^*}{\partial m}=\frac{\alpha A(\beta+\Delta)p[\beta(\beta+\Delta)(2k-g)-\alpha Bp]}{(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}$, $\frac{\partial x^*}{\partial m}=-\frac{A(\beta+\Delta)^2k[\beta(\beta+\Delta)(2k-g)-\alpha Bp]}{(1-\beta-\Delta)(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}$,

$\frac{\partial {\pi_p}^*}{\partial m}=-\frac{\alpha Ap[\beta(\beta+\Delta)(2k-g)-\alpha Bp]^2}{2\beta(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}$, and

$\frac{\partial {\pi_s}^*}{\partial m}=\frac{(1-\alpha)Ap(\alpha Ap+2\beta(\beta+\Delta)^2km)[\beta(\beta+\Delta)(2k-g)-\alpha Bp]}{4\beta(2\beta(\beta+\Delta)^2km-\alpha Ap)^2}\times[\frac{4\alpha Bp}{\alpha Ap+2\beta(\beta+\Delta)^2km}-\frac{\beta(\beta+\Delta)(2k-g)-\alpha Bp}{2\beta(\beta+\Delta)^2km-\alpha Ap}]$.

As $g_1<g<g_2$, we have $\beta(\beta+\Delta)(2k-g)-\alpha Bp>0$, and

$\frac{4\alpha Bp}{\alpha Ap+2\beta(\beta+\Delta)^2km}-\frac{\beta(\beta+\Delta)(2k-g)-\alpha Bp}{2\beta(\beta+\Delta)^2km-\alpha Ap}<-\frac{2[(\alpha Ap+2\beta(\beta+\Delta)^2km)\sqrt{m\eta A[q-p+\beta(1-q)]+m^2B^2}+Bm(2\beta(\beta+\Delta)^2km-\alpha Ap)]}{Am(\alpha Ap+2\beta(\beta+\Delta)^2km)}<0$,

thus $\frac{\partial x^*}{\partial m}>0$, $\frac{\partial x^*}{\partial m}<0$, $\frac{\partial {\pi_p}^*}{\partial m}<0$, and $\frac{\partial {\pi_s}^*}{\partial m}<0$.

C) From the optimal solution for $x^*=0$ and high r^* in Tables 2 and 3, we have:

$\frac{\partial x^*}{\partial m}=0$, $\frac{\partial r^*}{\partial m}=-\frac{(1-\alpha)(1-\beta-\Delta)p}{2(\beta+\Delta)m^2}<0$, $\frac{\partial {\pi_s}^*}{\partial m}=-\frac{(1-\alpha)^2(1-\beta-\Delta)^2p^2}{4(\beta+\Delta)^2m^2}<0$, $\frac{\partial {\pi_p}^*}{\partial m}=-\frac{(1-\alpha)\alpha(1-\beta-\Delta)^2p^2}{2(\beta+\Delta)^2m^2}<0$. □

Proof of Proposition 3: From the optimal solution for the medium x^* and r^* in Table 2, we have:

$\frac{\partial x^*}{\partial \Delta}=\frac{\alpha mp\left[2A(2\beta k(\beta+\Delta)-p\alpha B-\beta g(\beta+\Delta))+\beta(1-\beta-\Delta)(q-p)\left(2\beta k(\beta+\Delta)^2m-\alpha Ap\right)\right]}{(1-\beta-\Delta)\left(2\beta k(\beta+\Delta)^2m-\alpha Ap\right)^2}$,

$\frac{\partial r^*}{\partial \Delta}=\frac{A\left\{4\alpha\beta^2k(\beta+\Delta)mp(1-\beta-\Delta)(p-q)-\left(\alpha Ap+2\beta k(\beta+\Delta)^2m\right)[\beta(2k-g)+\alpha p((1-\beta-\eta)(p-q)+\beta\eta(1-q))]\right\}}{2\beta(1-\beta-\Delta)^2\left(2\beta k(\beta+\Delta)^2m-\alpha Ap\right)^2}$.

Let $g_x=\frac{2A(2\beta k(\beta+\Delta)-\alpha Bp)+\beta(1-\beta-\Delta)(q-p)\left(2\beta km(\beta+\Delta)^2-\alpha Ap\right)}{2A\beta(\beta+\Delta)}$,

$g_r = \frac{(2\beta k+\alpha p((1-\beta-\eta)(p-q)+\beta\eta(1-q)))\left(\alpha Ap+2\beta km(\beta+\Delta)^2\right)-4\alpha\beta^2 kmp(\beta+\Delta)(1-\beta-\Delta)(p-q)}{\beta\left(\alpha Ap+2\beta km(\beta+\Delta)^2\right)}$, then, $\frac{\partial x^*}{\partial \Delta} < 0$ if $g > g_x$, $\frac{\partial x^*}{\partial \Delta} \geq 0$ if $g \leq g_x$, $\frac{\partial r^*}{\partial \Delta} > 0$ if $g > g_r$, and $\frac{\partial r^*}{\partial \Delta} \leq 0$ if $g \leq g_r$.

As $g_1 \leq g \leq g_2$, by evaluating the difference between g_1, g_2. g_x, and g_r, we find that: (1) when $q > p$, it is easy to verify $g_x > g_2$, $g_r > g_2$, and $g_x > g_r$, thus, $\frac{\partial x^*}{\partial \Delta} > 0$ and $\frac{\partial r^*}{\partial \Delta} < 0$. (2) when $q \leq p$, it is easy to verify $g_x \leq g_r$, thus: (a) if $g_1 \leq g < g_x$, then $\frac{\partial x^*}{\partial \Delta} > 0$, $\frac{\partial r^*}{\partial \Delta} < 0$; (b) if $g_x \leq g \leq g_r$ then $\frac{\partial x^*}{\partial \Delta} \leq 0$, $\frac{\partial r^*}{\partial \Delta} \leq 0$; (c) if $g_r < g \leq g_2$, then $\frac{\partial x^*}{\partial \Delta} < 0$, $\frac{\partial r^*}{\partial \Delta} > 0$. □

Proof of Corollary 3: From the optimal solution for $x^* = 0$ and high r^* in Table 2, we have: $\frac{\partial r^*}{\partial \Delta} = -\frac{(1-\alpha)p}{2(\beta+\Delta)^2 m} < 0$. Thus, the higher the Δ, the lower the r^*. □

Proof of Proposition 4: By comparing the consumer welfare with and without manipulation, we have: $w_c(r, x \,|\, \mathbb{1}_r = 1) - w_c(r, x \,|\, \mathbb{1}_r = 0) = \frac{(1-x)\left\{\Delta\left[\beta(\beta+\Delta)(1-q)^2-(q-p)^2\right]-\beta(1-\beta-\Delta)^2 r^2\right\}}{2\beta(\beta+\Delta)}$. As $\Delta > 0$, $0 < \beta + \Delta < 1$ and $q \in (0, 1)$, thus (1) if $0 < \Delta < \frac{(q-p)^2-\beta^2(1-q)^2}{\beta(1-q)^2}$ then manipulation decrease consumer welfare for any given manipulation effort r; (2) otherwise if $\Delta \geq \frac{(q-p)^2-\beta^2(1-q)^2}{\beta(1-q)^2}$, then manipulation decrease consumer welfare when $r > \sqrt{\frac{\Delta\left[\beta(\beta+\Delta)(1-q)^2-(q-p)^2\right]}{\beta(1-\beta-\Delta)^2}}$, and increase consumer welfare when $r \leq \sqrt{\frac{\Delta\left[\beta(\beta+\Delta)(1-q)^2-(q-p)^2\right]}{\beta(1-\beta-\Delta)^2}}$. □

Proof of Proposition 5: From equation (8), we can get the Hessian matrix of $\pi_s(r, p \,|\, \mathbb{1}_r = 1, x = 0)$:

$$H_{rp} = \begin{bmatrix} \frac{\partial^2 \pi_s}{\partial r^2} & \frac{\partial^2 \pi_s}{\partial r \partial p} \\ \frac{\partial^2 \pi_s}{\partial p \partial r} & \frac{\partial^2 \pi_s}{\partial p^2} \end{bmatrix} = \begin{bmatrix} -2m & \frac{(1-\alpha)(1-\beta-\Delta)}{\beta+\Delta} \\ \frac{(1-\alpha)(1-\beta-\Delta)}{\beta+\Delta} & -\frac{2(1-\alpha)}{\beta+\Delta} \end{bmatrix} \tag{A7}$$

Thus, only when $4(\beta + \Delta)m - (1 - \alpha)(1 - \beta - \Delta)^2 \geq 0$, i.e., $m \geq \frac{(1-\alpha)(1-\beta-\Delta)^2}{4(\beta+\Delta)}$, is the Hessian matrix negative and $\pi_s(r, p \,|\, \mathbb{1}_r = 1, x = 0)$ concave around (r^*, p^*). Let $\frac{\partial \pi_s(r,p \,|\, \mathbb{1}_r=1,x=0)}{\partial r} = 0$ and $\frac{\partial \pi_s(r,p \,|\, \mathbb{1}_r=1,x=0)}{\partial p} = 0$, we have

$$r_1^* = \frac{(1-\alpha)(1-\beta-\Delta)(\beta+\Delta+(1-\beta-\Delta)q)}{4(\beta+\Delta)m-(1-\alpha)(1-\beta-\Delta)^2} \tag{A8}$$

$$p_1^* = \frac{2(\beta+\Delta)m(\beta+\Delta+(1-\beta-\Delta)q)}{4(\beta+\Delta)m-(1-\alpha)(1-\beta-\Delta)^2} \tag{A9}$$

Substituting equations (A8) and (A9) into (7) and (8), we have:

$$\pi_{s1}^{*} = \frac{(1-\alpha)m\,(\beta+\Delta+(1-\beta-\Delta)\,q)^{2}}{4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^{2}} \tag{A10}$$

$$\pi_{p1}^{*} = \frac{4\alpha(\beta+\Delta)m^{2}\,(\beta+\Delta+(1-\beta-\Delta)\,q)^{2}}{\left(4(\beta+\Delta)m-(1-\alpha)\,(1-\beta-\Delta)^{2}\right)^{2}} - g \tag{A11}$$

□

Proof of Corollary 4: From equation (8), we can get $\frac{\partial^2 \pi_s(p\,|\mathbb{I}_r=0,x=0)}{\partial p^2} = -\frac{2(1-\alpha)}{\beta} < 0$, which means that $\pi_s(p\,|\mathbb{I}_r = 0, x = 0)$ is a concave function of p^*. Let $\frac{\partial \pi_s(p\,|\mathbb{I}_r=0,x=0)}{\partial p} = 0$, we have

$$p_2^{*} = \frac{1}{2}(\beta+(1-\beta)q) \tag{A12}$$

Substituting equation (A12) into (7) and (8), we have:

$$\pi_{s2}^{*} = \frac{(1-\alpha)(\beta+(1-\beta)q)^2}{4\beta} \tag{A13}$$

$$\pi_{p2}^{*} = \frac{\alpha(\beta+(1-\beta)q)^2}{4\beta} \tag{A14}$$

By evaluating the difference in π_{s1}^* and π_{s2}^*, we find that:

$\pi_{s1} - \pi_{s2} = \frac{(1-\alpha)\left\{4m\Delta\left[\beta(\beta+\Delta)(1-q)^2-q^2\right]+(1-\alpha)(1-\beta-\Delta)^2[\beta+(1-\beta)q]^2\right\}}{4\beta\left[4(\beta+\Delta)m-(1-\alpha)(1-\beta-\Delta)^2\right]}$.

This means that only if $4m\Delta\left[\beta(\beta+\Delta)(1-q)^2-q^2\right]+(1-\alpha)\,(1-\beta-\Delta)^2\,[\beta+(1-\beta)q]^2 > 0$ does the seller manipulate online reviews, and then $p^* = p_1^*$, $r^* = r_1^*$; otherwise, the seller does not manipulate, and then $p^* = p_2^*$, $r^* = 0$. Simplify the constraint, let $q_2 = \frac{\sqrt{\beta(\beta+\Delta)}}{1+\sqrt{\beta(\beta+\Delta)}}$, $m_2 = \frac{(1-\alpha)(1-\beta-\Delta)^2[\beta+(1-\beta)q]^2}{4\Delta\left[q^2-\beta(\beta+\Delta)(1-q)^2\right]}$, then when $q \le q_2$ the seller manipulates review for any given m; when $q > q_2$ the seller manipulates review if $m \le m_2$, not manipulates reviews if $m > m_2$. □

Proof of Proposition 6: From equation (8), we can get the Hessian matrix of $\pi_s(r, p\,|\mathbb{I}_r = 1, x)$:

$$H_{rp} = \begin{bmatrix} \frac{\partial^2\pi_s}{\partial r^2} & \frac{\partial^2\pi_s}{\partial r\partial p} \\ \frac{\partial^2\pi_s}{\partial p\partial r} & \frac{\partial^2\pi_s}{\partial p^2} \end{bmatrix} = \begin{bmatrix} -2m & \frac{(1-\alpha)(1-\beta-\Delta)(1-x)}{\beta+\Delta} \\ \frac{(1-\alpha)(1-\beta-\Delta)(1-x)}{\beta+\Delta} & -2(1-\alpha)\left(\frac{(1-\eta)x}{\beta}+\frac{1-x}{\beta+\Delta}\right) \end{bmatrix} \tag{A15}$$

Thus, only when $4(\beta + \Delta)m\,[(\beta + \Delta)(1 - \eta)x + \beta(1 - x)] - (1 - \alpha)\beta\,(1 - \beta - \Delta)^2(1 - x)^2 > 0$ is the Hessian matrix negative and $\pi_s(r, p\,|\mathbb{I}_r = 1, x)$ concave around $(r(x), p(x))$. The seller's best response computed from $\partial\pi_s/\partial r = 0$ and $\partial\pi_s/\partial p = 0$.

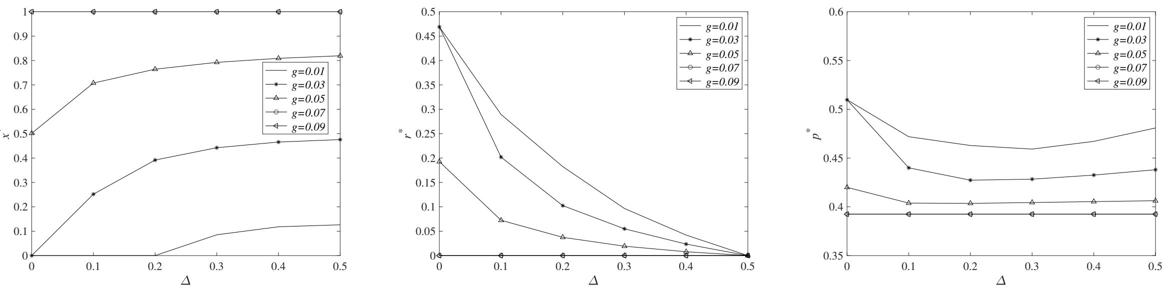
Proof of Corollary 6: By taking the first-order directive of $p(x)$ and $r(x)$, we have:

$$\frac{dp(x)}{dx} = -\frac{8m^2\beta(\beta+\Delta)^3\Delta(1-\eta)(1-q)+2m\beta(\beta+\Delta)(1-\alpha)(1-\beta-\Delta)^2(1-x)[(\beta+\Delta)\beta(1-q)(2-\eta-\eta x)+(\beta+\Delta)q(1-\eta)(x+1)+\beta q(1-x)]}{\left\{4(\beta+\Delta)m[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2\right\}^2},$$

$$\frac{dr(x)}{dx} = -\frac{(\beta+\Delta)(1-\alpha)(1-\beta-\Delta)\left(8mqx\beta(\beta+\Delta)(1-\eta)(1-x)+\beta(1-\alpha)(1-\beta-\Delta)^2(1-\eta)(1-x)^2(\beta-\beta q+q)\right)}{\left\{4(\beta+\Delta)m[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2\right\}^2}$$
$$-\frac{4m(\beta+\Delta)(1-\alpha)(1-\beta-\Delta)\left\{(\beta+\Delta)^2\beta(1-\eta)(1-q)\left(1-\eta x^2\right)+qx^2(\beta+\Delta)^2(1-\eta)^2+\beta^2(1-x)^2[(\beta+\Delta)\eta(1-q)+q]\right\}}{\left\{4(\beta+\Delta)m[(\beta+\Delta)(1-\eta)x+\beta(1-x)]-(1-\alpha)\beta(1-\beta-\Delta)^2(1-x)^2\right\}^2}.$$

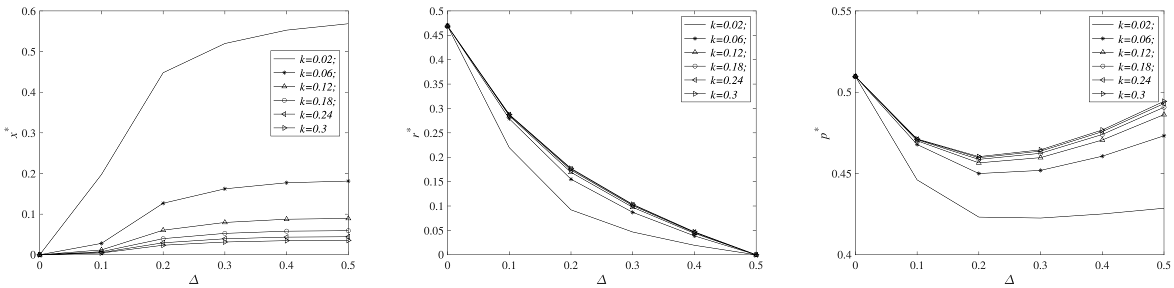
As $q, \alpha \in (0, 1)$, $1 > (\beta + \Delta) > \beta > 0$, $0 < \eta \leq 1$, $0 \leq x \leq 1$, then $\Delta > 0$, $1 - q > 0$, $1 - \alpha > 0$, $1 - \eta \geq 0$, $1 - x \geq 0$, $2 - \eta - \eta x \geq 0$, $1 - \beta - \Delta > 0$, $1 - \eta x^2 \geq 0$, thus, $\frac{dp(x)}{dx} \leq 0$ and $\frac{dr(x)}{dx} \leq 0$. □

Appendix C. Sensitivity Analysis



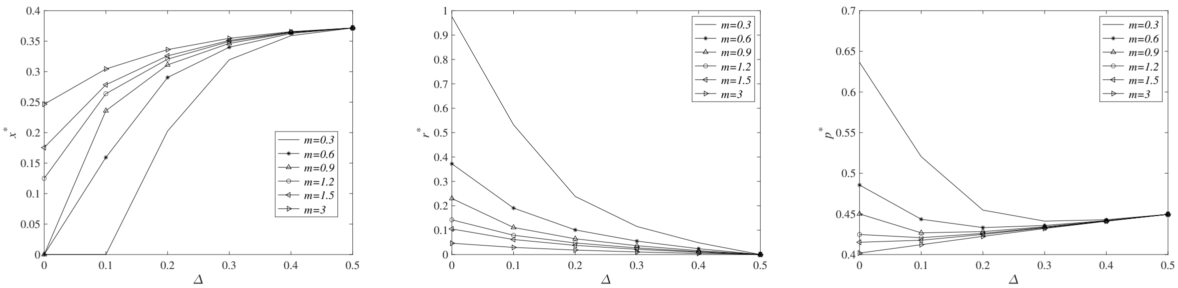
(a) (b) (c)

Figure C1: The impact of Δ on optimal solutions with different g ($\eta = 0.25$, $k = 0.03$, $m = 0.5$, $q = 0.57$)



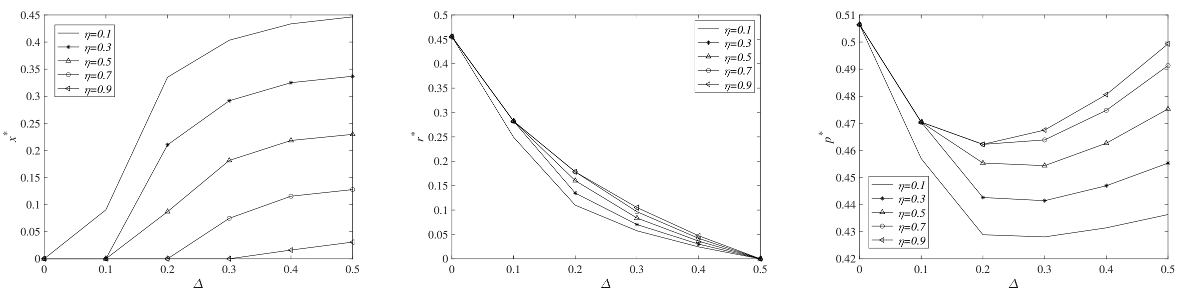
(a) (b) (c)

Figure C2: The impact of Δ on optimal solutions with different k ($\eta = 0.25$, $q = 0.57$, $m = 0.5$, $g = 0.024$)



(a) (b) (c)

Figure C3: The impact of Δ on optimal solutions with different m ($\eta = 0.25$, $k = 0.03$, $q = 0.57$, $g = 0.024$)



(a) (b) (c)

Figure C4: The impact of Δ on optimal solutions with different η ($q = 0.57$, $k = 0.03$, $m = 0.5$, $g = 0.024$)